

1. HODGE DUAL IN MINKOWSKI SPACE

4-dimensional Minkowski space has an orthonormal, oriented basis of 1-forms given by

$$\{dx, dy, dz, dt\}$$

with $g(dt, dt) = -1$, $g(dx, dx) = g(dy, dy) = g(dz, dz) = 1$, and all others zero. The “volume element” (choice of orientation) is given by $\omega = dx \wedge dy \wedge dz \wedge dt$.

- (a) Determine the Hodge dual operator $*$ on all forms by computing its action on basis forms at each rank.
- (b) How does your answer change if the opposite orientation is chosen, namely

$$\omega = dt \wedge dx \wedge dy \wedge dz$$