1. **SPHERICAL COORDINATES**

Consider spherical coordinates in 3-dimensional Euclidean space with the usual orientation, namely $\omega = r^2 \sin\theta \, dr \wedge d\theta \wedge d\phi$.

WARNING: These are "physics" conventions: θ is the angle from the north pole (colatitude), and ϕ is the angle in the xy-plane (longitude).

(a) Determine the Hodge dual operator $*$ on all forms (expressed in spherical coordinates) by computing its action on basis forms at each rank.

(b) Compute the dot and cross products of 2 generic "vector fields" (really 1-forms) in spherical coordinates using the expressions:

$$\alpha \cdot \beta = *(\alpha \wedge *\beta)$$
$$\alpha \times \beta = *(\alpha \wedge \beta)$$

You may express your results either with respect to an orthonormal basis or with respect to a "coordinate" (non-orthonormal) spherical basis; make sure you know which you're doing. ("Generic" means you must give an answer valid for ***any*** *2 vectors.)*