Applications

$\alpha \in \Lambda^{n-1} \Longrightarrow d\alpha \in \Lambda^n$

$\Longrightarrow d\alpha = f\omega$ for some f

$$\therefore\ 0 = \int_S d\alpha = \int_S f\omega$$

$\Longrightarrow f = 0$ somewhere

$\Longrightarrow d\alpha = 0$ somewhere

Ex: f on circle must satisfy
$df = 0$ somewhere
($\vec{\nabla} f$ must point radially somewhere)

Ex: F on sphere must satisfy
$dF = 0$ somewhere
($\vec{\nabla} \times \vec{F}$ must point radially somewhere)

Topology

§20.4

$$\boxed{\begin{array}{ll} \alpha \ \underline{\text{closed}} \iff d\alpha = 0 \\ \alpha \ \underline{\text{exact}} \iff \exists \beta : \alpha = d\beta \end{array}}$$

β is a <u>potential</u> for α

<u>Poincaré Lemma</u>: exact $\Rightarrow$ closed

<u>Pf</u>: $d^2 = 0$

<u>Ex</u>: $\vec{\nabla} \times \vec{\nabla} f = \vec{0}$

$\vec{\nabla} \cdot \vec{\nabla} \times \vec{F} = 0$

<u>Converse</u>: If $D \subset \mathbb{R}^n$ is <u>contractible</u> ("no holes")

then closed $\Rightarrow$ exact <u>on D</u>

<u>Ex</u>: $D = \mathbb{R}^3$

$\vec{\nabla} \times \vec{E} = \vec{0} \Longrightarrow \exists \Phi : \vec{E} = -\vec{\nabla} \Phi$

$\vec{\nabla} \cdot \vec{B} = 0 \Longrightarrow \exists \vec{A} : \vec{B} = \vec{\nabla} \times \vec{A}$

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potential not unique:

$$d\beta = d(\beta + d\gamma)$$

(generalizes $df = d(f + C)$)

$$\alpha \in \Lambda^p \Rightarrow \beta \in \Lambda^{p-1}, \ \gamma \in \Lambda^{p-2}$$

<u>Ex</u>: $\oint d(\sin\phi) = \sin\phi \Big|_0^{2\pi} = 0$

$\oint d\phi = 2\pi \neq 0$

$\Rightarrow$ "$d\phi$" $\neq df$ for any (single, well-def) function f

$\Rightarrow$ $d\phi$ closed but not exact TOPOLOGY!