Pictures of Differential Forms

Tensors $\quad df(\vec{v}) = \vec{v}(f) = \vec{\nabla} f \cdot \vec{v}$

Ex: $dx = \vec{\nabla} x \cdot d\vec{r} = \hat{x} \cdot d\vec{r}$

$\Longrightarrow dx(\vec{v}) = \hat{x}(\vec{v}) = v_x$

How can we represent this operation?

(show pictures here)

df is small because we are zoomed in at a point (calculus)

df is large because at that point it behaves like a finite vector (linear algebra)

Linear Maps

$$A : \Lambda^1 \longrightarrow \Lambda^1$$
$$\alpha \longmapsto A(\alpha)$$

<u>linear</u>: $\boxed{A(f\alpha + \beta) = fA(\alpha) + A(\beta)}$

<u>basis</u>: $A(dx^i) = a^i{}_j \, dx^j$

$$A \longleftrightarrow (a^i{}_j)$$

$$A : \Lambda^p \longrightarrow \Lambda^p$$
$$\alpha \wedge \beta \longmapsto A\alpha \wedge A\beta$$

$$\therefore \omega \in \Lambda^n \Rightarrow A\omega = \lambda \omega$$
$$=: |A| \omega$$

Basis independent definition of determinant!

$$\therefore |AB| \omega = (AB)\omega$$
$$= A(B\omega)$$
$$= A(|B|\omega)$$
$$= |A||B| \omega$$

$$\therefore \boxed{|AB| = |A||B|}$$