Example: Find the work done by $\vec{F}$ along (part of) the x-axis

$$\int_I \vec{F}\cdot d\vec{r} = \int_I \vec{F}\cdot dx\,\hat{x} = \int_I F_x\,dx$$

Use what you know:

$$d\vec{r} = dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}$$

but y, z const $\Rightarrow dy = 0 = dz$

$$F = \vec{F}\cdot d\vec{r}$$

$$\Rightarrow \int_I \vec{F}\cdot d\vec{r} = \int_I F$$

Example: Find the flux of $\vec{F}$ up through (part of) the xy-plane

$$\int_R \vec{F}\cdot d\vec{A} = \int_R \vec{F}\cdot \hat{z}\, dx\, dy = \int_R F_z\, dx\, dy$$

Use what you know:

$$d\vec{A} = d\vec{r}_1 \times d\vec{r}_2 = dx\,\hat{x} \times dy\,\hat{y} = dx\,dy\,\hat{z}$$

$$*F = \vec{F}\cdot d\vec{A}$$

$$\Rightarrow \int_R \vec{F}\cdot d\vec{A} = \int_R *F$$

Idea $\int f\, dx\wedge dy := \pm \int f\, dx\, dy$ ← depends on orientation