Differential forms in 3d

0 $f = f\,1$

1 $F = \vec{F}\cdot d\vec{r} = F_x\,dx + F_y\,dy + F_z\,dz$

2 $\alpha = \vec{F}\cdot d\vec{A} = F_x\,dy\wedge dz + F_y\,dz\wedge dx + F_z\,dx\wedge dy$

3 $\beta = f\,dV = f\,dx\wedge dy\wedge dz$

Bases

0 1

1 dx, dy, dz

2 $dy\wedge dz, dz\wedge dx, dx\wedge dy$

3 $dx\wedge dy\wedge dz$

Duality

0 $f \overset{*}{\longleftrightarrow} f\,dV$ 3

1 $\vec{F}\cdot d\vec{r} \overset{*}{\longleftrightarrow} \vec{F}\cdot d\vec{A}$ 2

$\longmapsto$

$$\boxed{\begin{aligned} *f &= f\,dV \\ *F &= \vec{F}\cdot d\vec{A} \end{aligned}}$$

Theorems

Divergence $$\oint_{\partial V} \vec{F}\cdot d\vec{A} = \int_V \vec{\nabla}\cdot\vec{F}\, dV$$

Stokes $$\oint_{\partial S} \vec{F}\cdot d\vec{r} = \int_S \vec{\nabla}\times\vec{F}\cdot d\vec{A}$$

FTC $$\oint_{\partial C} f = \int_C df = \int_C \vec{\nabla}f\cdot d\vec{r}$$

$$\boxed{\begin{aligned} d(f) &= df = \vec{\nabla}f\cdot d\vec{r} \\ d(\vec{F}\cdot d\vec{r}) &= \vec{\nabla}\times\vec{F}\cdot d\vec{A} \\ d(\vec{F}\cdot d\vec{A}) &= \vec{\nabla}\cdot\vec{F}\, dV \end{aligned}}$$

exterior derivative

SUMMARY

0-forms	f	$\langle 1 \rangle$
1-forms	$\vec{F}\cdot d\vec{r}$	$\langle dx, dy, dz \rangle$
2-forms	$\vec{F}\cdot d\vec{A}$	$\langle dy\wedge dz, dz\wedge dx, dx\wedge dy \rangle$
3-forms	$f\,dV$	$\langle dx\wedge dy\wedge dz \rangle$

$$dy\wedge dx = -dx\wedge dy$$

$$*dx = dy\wedge dz$$
$$*1 = dx\wedge dy\wedge dz$$

$$F = \vec{F}\cdot d\vec{r}$$
$$G = \vec{G}\cdot d\vec{r}$$

$$\Longrightarrow \quad F\wedge G = \vec{F}\times\vec{G}\cdot d\vec{A}$$
$$*(F\wedge G) = \vec{F}\times\vec{G}\cdot d\vec{r}$$
$$F\wedge *G = \vec{F}\cdot\vec{G}\,dV$$
$$*(F\wedge *G) = \vec{F}\cdot\vec{G}$$