Name ____

MTH 434/534 **MIDTERM** Winter 2020

*Work on the paper provided, turn in this page as well and put your name on each page. Feel free to ask questions during the exam — I will answer any I feel are appropriate. Show your work! It is to your advantage to turn in all your scratch work (clearly labeled!). In particular, it is to your advantage **not** to erase anything.*
You should attempt every problem. If you are unable to complete a calculation, set it up and give a clear description of what you would have done for possible partial credit. Do not waste time on unnecessary simplifications, which will not substantially improve your grade.

You may wish to recall the following facts:

$$p = (p_1, p_2, p_3), \quad x_i = x, y, z, \quad U_1(p) = (1,0,0)_p,\ U_2(p) = (0,1,0)_p,\ U_3(p) = (0,0,1)_p$$

$$\vec{\boldsymbol{v}} = v_i U_i = v_i \hat{\boldsymbol{u}}_i = v_x \hat{\boldsymbol{x}} + v_y \hat{\boldsymbol{y}} + v_z \hat{\boldsymbol{z}}, \quad \vec{\boldsymbol{v}}[f] = v_i \frac{\partial f}{\partial x_i}, \quad \vec{\boldsymbol{v}}[x] = v_x,\ \vec{\boldsymbol{v}}[y] = v_y,\ \vec{\boldsymbol{v}}[z] = v_z$$

$$\alpha(u) = (\alpha_1(u), \alpha_2(u), \alpha_3(u)), \quad x = \alpha_1(u),\ y = \alpha_2(u),\ z = \alpha_3(u)$$

$$\vec{\boldsymbol{v}} = \left(\frac{d\alpha}{du}\right)_{\alpha(u)}, \quad \vec{\boldsymbol{a}} = \frac{d\vec{\boldsymbol{v}}}{du} = \left(\frac{d^2\alpha}{du^2}\right)_{\alpha(u)}, \quad \vec{\boldsymbol{\jmath}} = \frac{d\vec{\boldsymbol{a}}}{du} = \left(\frac{d^3\alpha}{du^3}\right)_{\alpha(u)}$$

$$v = |\vec{\boldsymbol{v}}| = \|\vec{\boldsymbol{v}}\| = \sqrt{\vec{\boldsymbol{v}} \cdot \vec{\boldsymbol{v}}}, \quad s = \int v\, du = h(u), \quad \alpha(u) = \beta(s) \Longrightarrow \left|\frac{d\beta}{ds}\right| = \frac{|d\alpha/du|}{ds/du} = 1$$

$$\vec{\boldsymbol{v}} = v\hat{\boldsymbol{T}}, \quad \vec{\boldsymbol{a}} = \frac{dv}{du}\hat{\boldsymbol{T}} + \kappa v^2 \hat{\boldsymbol{N}}\ (\kappa \geq 0), \quad \vec{\boldsymbol{B}} = \hat{\boldsymbol{T}} \times \hat{\boldsymbol{N}}$$

$$\frac{d\hat{\boldsymbol{T}}}{du} = \kappa v \hat{\boldsymbol{N}}, \quad \frac{d\hat{\boldsymbol{N}}}{du} = -\kappa v \hat{\boldsymbol{T}} + \tau v \vec{\boldsymbol{B}}, \quad \frac{d\vec{\boldsymbol{B}}}{du} = -\tau v \hat{\boldsymbol{N}}$$

$$\kappa = \frac{|\vec{\boldsymbol{v}} \times \vec{\boldsymbol{a}}|}{v^3}, \quad \tau = \frac{(\vec{\boldsymbol{v}} \times \vec{\boldsymbol{a}}) \cdot \vec{\boldsymbol{\jmath}}}{\kappa^2 v^6}, \qquad \psi = \psi_i dx_i = \psi_x dx + \psi_y dy + \psi_z dz$$

$$df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial z}dz, \quad df(\vec{\boldsymbol{v}}) = \vec{\boldsymbol{v}}[f] = v_i \frac{\partial f}{\partial x_i}, \quad dx(\vec{\boldsymbol{v}}) = v_x,\ dy(\vec{\boldsymbol{v}}) = v_y,\ dz(\vec{\boldsymbol{v}}) = v_z$$

$$dy \wedge dx = -dx \wedge dy, \quad d(f\,dx) = df \wedge dx, \quad d(d\psi) = 0, \quad d(\psi \wedge \eta) = d\psi \wedge \eta \pm \psi \wedge d\eta$$

$$*1 = dx \wedge dy \wedge dz, \quad *dx = dy \wedge dz,\ *dy = dz \wedge dx,\ *dz = dx \wedge dy, \quad ** = 1$$

$$\hat{\boldsymbol{e}}_i \cdot \hat{\boldsymbol{e}}_j = \delta_{ij}, \quad \hat{\boldsymbol{e}}_i = a_{ij} \hat{\boldsymbol{u}}_j\ (\Rightarrow \hat{\boldsymbol{u}}_j = a_{kj} \hat{\boldsymbol{e}}_k), \quad A = (a_{ij}), \quad A^T = A^{-1}$$

$$\vec{\boldsymbol{w}} = w_i\, \hat{\boldsymbol{u}}_i \Rightarrow \nabla_{\vec{\boldsymbol{v}}}\, \vec{\boldsymbol{w}} = \vec{\boldsymbol{v}}[w_i]\, \hat{\boldsymbol{u}}_i, \quad \nabla_{\vec{\boldsymbol{v}}}(f\vec{\boldsymbol{w}}) = (\nabla_{\vec{\boldsymbol{v}}} f)\, \vec{\boldsymbol{w}} + f\, \nabla_{\vec{\boldsymbol{v}}}\, \vec{\boldsymbol{w}} = \vec{\boldsymbol{v}}[f]\, \vec{\boldsymbol{w}} + f\, \nabla_{\vec{\boldsymbol{v}}}\, \vec{\boldsymbol{w}}$$

$$\nabla_{\vec{\boldsymbol{v}}}\, \hat{\boldsymbol{e}}_i = \omega_{ij}(\vec{\boldsymbol{v}})\, \hat{\boldsymbol{e}}_j \Longrightarrow \omega_{ij}(\vec{\boldsymbol{v}}) = \nabla_{\vec{\boldsymbol{v}}}\, \hat{\boldsymbol{e}}_i \cdot \hat{\boldsymbol{e}}_j = -\omega_{ji}(\vec{\boldsymbol{v}})$$

$$\nabla_{\vec{\boldsymbol{v}}}\, \hat{\boldsymbol{e}}_i = \nabla_{\vec{\boldsymbol{v}}}\, (a_{ij}\, \hat{\boldsymbol{u}}_j) = \vec{\boldsymbol{v}}[a_{ij}]\, \hat{\boldsymbol{u}}_j = da_{ij}(\vec{\boldsymbol{v}})\, a_{kj}\, \hat{\boldsymbol{e}}_k \Rightarrow (\omega_{ij}) = dA\, A^T$$

$$\sigma_i(\hat{\boldsymbol{e}}_j) = \delta_{ij}, \quad \hat{\boldsymbol{e}}_j = a_{jk} \hat{\boldsymbol{u}}_k \Rightarrow \sigma_i = a_{ik}\, dx_k$$

$$d\sigma_i = \omega_{ij} \wedge \sigma_j \quad d\omega_{ij} = \omega_{ik} \wedge \omega_{kj}$$

(Repeated indices are summed over unless explicitly stated otherwise.)