

1. DECOMPOSABLE FORMS

Denote the p -forms in \mathbb{R}^n by $\Lambda^p(\mathbb{R}^n)$. A typical 1-form in \mathbb{R}^2 would therefore take the form $F = F_x dx + F_y dy \in \Lambda^1(\mathbb{R}^2)$.

A p -form $\beta \in \Lambda^p(\mathbb{R}^n)$ is called *decomposable* if there exist 1-forms $\alpha_i \in \Lambda^1(\mathbb{R}^n)$ with

$$\beta = \alpha_1 \wedge \dots \wedge \alpha_p$$

- (a) Show that all elements of $\Lambda^2(\mathbb{R}^3)$, that is, all 2-forms in \mathbb{R}^3 , are decomposable. In other words, show that

$$H = H_x dy \wedge dz + H_y dz \wedge dx + H_z dx \wedge dy$$

is decomposable.

HINT: Consider the previous assignment!

You may cite your solution to the previous assignment without proof, so long as an explicit reference is given (“see HW #1”). If you do this, it wouldn’t hurt to include a copy of your previous assignment.

- (b) Find an example of an *indecomposable* 2-form $\gamma \in \Lambda^2(\mathbb{R}^n)$.

HINT: Don’t work in \mathbb{R}^3 ...

- (c) Is $\gamma \wedge \gamma = 0$? Should it be? Can it be?

2. PICTURES OF FORMS

Let $\alpha = 3 dx$ and $\beta = 4 dy$.

- (a) Draw a single picture showing the “stacks” corresponding to both α and β .

You may want to use different colors for the stacks corresponding to α and β . Your drawing should be correctly scaled.

- (b) Draw a separate picture showing the stack corresponding to $\gamma = \alpha + \beta$.

- (c) Choose a vector $\vec{v} \in \mathbb{R}^2$ that is *not* parallel to the coordinate axes. Add \vec{v} to your previous diagrams and use them to compute $\alpha(\vec{v})$, $\beta(\vec{v})$, and $\gamma(\vec{v})$.

Your computation should be geometric, not algebraic.

- (d) Did you obtain $\gamma(\vec{v}) = \alpha(\vec{v}) + \beta(\vec{v})$? Should you have?