SOME ORTHOGONAL COORDINATE SYSTEMS IN $\mathbb{R}^3$

RECTANGULAR:

$$ds^2 = dx^2 + dy^2 + dz^2$$
$$\omega = dx \wedge dy \wedge dz$$

CYLINDRICAL:

$$x = r\cos\phi; \quad y = r\sin\phi; \quad z = z$$
$$ds^2 = dr^2 + r^2\, d\phi^2 + dz^2$$
$$\omega = r\, dr \wedge d\phi \wedge dz$$

SPHERICAL:

$$x = r\sin\theta\cos\phi; \quad y = r\sin\theta\sin\phi; \quad z = r\cos\theta$$
$$ds^2 = dr^2 + r^2\, d\theta^2 + r^2\sin^2\theta\, d\phi^2$$
$$\omega = r^2\sin\theta\, dr \wedge d\theta \wedge d\phi$$

PARABOLIC CYLINDRICAL:

$$x = \frac{1}{2}(u^2 - v^2); \quad y = uv; \quad z = z$$

PARABOLOIDAL:

$$x = uv\cos\phi; \quad y = uv\sin\phi; \quad z = \frac{1}{2}(u^2 - v^2)$$

ELLIPTIC CYLINDRICAL:

$$x = a\cosh u\cos v; \quad y = a\sinh u\sin v; \quad z = z$$

PROLATE SPHEROIDAL:

$$x = a\sinh\xi\sin\eta\cos\phi; \quad y = a\sinh\xi\sin\eta\sin\phi; \quad z = a\cosh\xi\cos\eta$$

OBLATE SPHEROIDAL:

$$x = a\cosh\xi\cos\eta\cos\phi; \quad y = a\cosh\xi\cos\eta\sin\phi; \quad z = a\sinh\xi\sin\eta$$

BIPOLAR:

$$x = \frac{a\sinh v}{\cosh v - \cos u}; \quad y = \frac{a\sin u}{\cosh v - \cos u}; \quad z = z$$

TOROIDAL:

$$x = \frac{a\sinh v\cos\phi}{\cosh v - \cos u}; \quad y = \frac{a\sinh v\sin\phi}{\cosh v - \cos u}; \quad z = \frac{a\sin u}{\cosh v - \cos u}$$