MTH 434/534 **HW #3** Winter 2015

INNER PRODUCTS

1. Consider the inner product defined by

$$\vec{\boldsymbol{v}} \star \vec{\boldsymbol{w}} = \frac{1}{2}\left(v^x w^y + v^y w^x\right)$$

where $\vec{\boldsymbol{v}} = v^x\,\hat{\boldsymbol{x}} + v^y\,\hat{\boldsymbol{y}}$ and $\vec{\boldsymbol{w}} = w^x\,\hat{\boldsymbol{x}} + w^y\,\hat{\boldsymbol{y}}$ are ordinary vectors in $\mathbb{R}^2$.

(a) Compute $\vec{\boldsymbol{v}} \star \vec{\boldsymbol{v}}$.

(b) Find a vector $\vec{\boldsymbol{u}} \in \mathbb{R}^2$ such that $\vec{\boldsymbol{u}} \star \vec{\boldsymbol{u}} = 1$

(c) Find a vector $\vec{\boldsymbol{v}} \in \mathbb{R}^2$ such that $\vec{\boldsymbol{v}} \star \vec{\boldsymbol{v}} = -1$

(d) How many independent vectors $\vec{\boldsymbol{w}} \in \mathbb{R}^2$ are there satisfying $\vec{\boldsymbol{w}} \star \vec{\boldsymbol{w}} = 0$?

2. Now work in $\mathbb{R}^3$, and suppose there is an inner product such that

$$\vec{\boldsymbol{v}} \star \vec{\boldsymbol{v}} = v^x v^y + v^y v^z + v^z v^x$$

for *any* vector $\vec{\boldsymbol{v}} = v^x\,\hat{\boldsymbol{x}} + v^y\,\hat{\boldsymbol{y}} + v^z\,\hat{\boldsymbol{z}}$ in $\mathbb{R}^3$.

(a) Determine $\vec{\boldsymbol{v}} \star \vec{\boldsymbol{w}}$.

(b) Find a basis of $\mathbb{R}^3$ such that each basis vector $\vec{\boldsymbol{u}}$ satisfies either $\vec{\boldsymbol{u}} \star \vec{\boldsymbol{u}} = 1$ or $\vec{\boldsymbol{u}} \star \vec{\boldsymbol{u}} = -1$, and any two basis vectors $\vec{\boldsymbol{u}}$, $\vec{\boldsymbol{v}}$ satisfy $\vec{\boldsymbol{u}} \star \vec{\boldsymbol{v}} = 0$.

EXTRA CREDIT:

Using the above results, or otherwise, rewrite xy as the sum or difference of exactly two squares, and $xy + yz + zx$ as the sum or difference of exactly three squares.

HINT: Your previous answers should help.