

1. **ORTHOGONAL COORDINATES**

Choose any orthogonal coordinate system in 3-dimensional Euclidean space \mathbb{R}^3 other than Cartesian, cylindrical, or spherical coordinates. (You may see me for suggestions.) Working in an orthonormal basis, compute the gradient and Laplacian of an arbitrary function, and the curl and divergence of an arbitrary “vector field” (again, really a 1-form), using the expressions:

$$\nabla f = df$$

$$\nabla \times \alpha = *d\alpha$$

$$\nabla \cdot \alpha = *d*\alpha$$

$$\Delta f = \nabla \cdot \nabla f = *d*df$$

You may check your answer in standard reference books, but you should use exterior differentiation and Hodge duality in your computation.