DECOMPOSABLE FORMS

Denote the p-forms in $\mathbb{R}^n$ by $\bigwedge^p(\mathbb{R}^n)$. A typical 1-form in $\mathbb{R}^2$ would therefore take the form $F = F_x\,dx + F_y\,dy \in \bigwedge^1(\mathbb{R}^2)$.

A p-form $\beta \in \bigwedge^p(\mathbb{R}^n)$ is called *decomposable* if there exist 1-forms $\alpha_i \in \bigwedge^1(\mathbb{R}^n)$ with

$$\beta = \alpha_1 \wedge ... \wedge \alpha_p$$

1. Show that all elements of $\bigwedge^2(\mathbb{R}^3)$, that is, all 2-forms in $\mathbb{R}^3$, are decomposable. In other words, show that

$$H = H_x\,dy \wedge dz + H_y\,dz \wedge dx + H_z\,dx \wedge dy$$

is decomposable.

HINT: Consider the previous assignment!

You may cite your solution to the previous assignment without proof, so long as an explicit reference is given ("see HW #1"). If you do this, it wouldn't hurt to include a copy of your previous assignment.

2. Find an example of an *indecomposable* 2-form.

HINT: Don't work in $\mathbb{R}^3$*...*

EXTRA CREDIT:

- Show that all 3-forms are decomposable in $\mathbb{R}^4$.
- Can you argue that all elements of $\bigwedge^{n-1}(\mathbb{R}^n)$ are decomposable?