DECOMPOSABLE FORMS

Let V be a vector space. A p-vector $\beta \in \bigwedge^p V$ is called *decomposable* if and only if there exist vectors $\alpha_i \in V$ with

$$\beta = \alpha_1 \wedge ... \wedge \alpha_p$$

1. Let $\vec{\boldsymbol{u}}$ be an ordinary vector in $\mathbb{R}^3$, so that

$$\vec{\boldsymbol{u}} = A\,\hat{\boldsymbol{\imath}} + B\,\hat{\boldsymbol{\jmath}} + C\,\hat{\boldsymbol{k}}$$

for some constants A, B, C. Find two vectors $\vec{\boldsymbol{v}}$ and $\vec{\boldsymbol{w}}$ such that

$$\vec{\boldsymbol{u}} = \vec{\boldsymbol{v}} \times \vec{\boldsymbol{w}}$$

It is possible to solve this problem by brute force; find a better way if you can.
HINT: What properties should $\vec{\boldsymbol{v}}$ and $\vec{\boldsymbol{w}}$ have?

2. Show that if $\dim V = 3$ then all 2-vectors are decomposable. In other words, show that if $\{\rho, \sigma, \tau\}$ is a basis of V then

$$\gamma = A\,\sigma \wedge \tau + B\,\tau \wedge \rho + C\,\rho \wedge \sigma$$

is decomposable for any constants A, B, C.
HINT: Adapt your answer to the previous problem!

3. Find an example of an *indecomposable* 2-vector.
HINT: Don't work in $\mathbb{R}^3$...

EXTRA CREDIT:

- Show that if $\dim V = 4$ then all 3-vectors are decomposable.
- Can you argue that all $(n-1)$-vectors in dimension n are decomposable?