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WEDGE

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1. INTRODUCTION

This document describes how to use the *Mathematica* package for differential forms called WEDGE, which can calculate the three basic operations of $\wedge$ (exterior product), $*$ (Hodge dual), and d (exterior derivative). In its current implementation, an explicit set of orthogonal coordinates must be given.

2. GETTING STARTED

Start *Mathematica* (version 3). On UNIX machines, this is normally done via the command `mathematica &` for the notebook front end or `math` for the shell interface. On the OSU Math or Physics networks, load WEDGE by typing `<<~tevian/math/wedge.m` . On other systems, you will need to obtain a copy of the file `wedge.m` from me, install it in a suitable place on your system, and then load it with a command like `<<wedge.m` .

3. CHOOSING COORDINATES

Specify your coordinates with the command `SetCoords`. For instance, to use spherical coordinates, type `SetCoords[{r,θ,ϕ}]` . There are also internal variables q^i, which you can use instead by typing `SetCoords[n]` , where n is the desired number of coordinates. Please do not use q as one of your variables!

In the notebook front end, θ and ϕ can be entered directly as `\[Theta]` and `\[Phi]` , respectively. There are also aliases for these symbols involving the escape key ESC, which are easier to type, which in this case are ESC `q` ESC and ESC `f` ESC , respectively. These features are not available in the shell interface. You can of course give these variables other names if you wish.

You also need to specify the metric by defining the variable `LineElement` to be the line element. For instance, in spherical coordinates, type

```
LineElement = dr^2 + r^2 dθ^2 + r^2 Sin[θ]^2 dϕ^2
```

In the notebook front end, exponents can also be entered directly using the control key CTRL. For instance, instead of `r^2`, you can type `r` CTRL-`6` `2` , where CTRL-`6` means to type `6` while holding down the control key. (Note that `^` is an "upper case 6", which is typed as SHIFT-`6`.)

4. COMPUTATION

There are 3 predefined 1-forms called α, β, and γ, and you may of course define your own. Use ordinary *Mathematica* syntax; the (coordinate) basis 1-forms are obtained by simply prepending a "`d`" to your variable names. Wedge products are computed using the command `wedge` (which can take more than 2 arguments), Hodge duals using the command `star` , and exterior derivatives using the command `d` . For instance, try the commands:

```
Simplify[star[wedge[\[Alpha],\[Beta]]]]
Factor[star[wedge[\[Alpha],star[\[Beta]]]]]
Simplify[star[d[\[Alpha]]]]
Simplify[star[d[star[\[Alpha]]]]]
```

In the notebook front end, you can also enter "wedge" and "star" directly from the keyboard using `\[Wedge]` and `\[Null]\[Star]`, respectively. (The "Null" character is not needed to obtain the *symbol* $*$, but rather to overcome *Mathematica*'s insistence that $*$ is a *binary* rather than a *unary* operator.) Please note that, unlike the spelled-out versions, these operators will *not* recognize the gradients of your chosen coordinates as 1-forms unless you define this explicitly. For spherical coordinates, this is done by typing `{dr, dθ, dϕ} = {dq^1, dq^2, dq^3}`. Do this and try the commands `\[Null]\[Star]dr` and `dr\[Wedge]dr`. For more complicated expressions, you may wish to use the command `WCollect` to collect the terms using an appropriate basis, for instance `WCollect[\[Null]\[Star]\[Alpha]]`.