MTH 434/534 **HW #6** Winter 2007

1. **VECTOR POTENTIALS**

Consider the 2-form $\beta = 2yz\,dy \wedge dz + 2xz\,dz \wedge dx + 2xy\,dx \wedge dy$.

(a) Is β *closed*, that is, does $d\beta = 0$?

(b) Is β *exact*, that is, does there exist a 1-form α such that $d\alpha = \beta$?
If β is not exact, explain why. If β is exact, find the most general solution α.

(c) What problem in vector calculus have you solved?

2. **INTEGRATION ON THE SPHERE**

(a) Choose a particular 1-form β in $\mathbb{R}^3$. Compute $\alpha = d\beta$. Show that

$$\int_{\mathbb{S}^2} \alpha = 0$$

where $\mathbb{S}^2$ denotes the unit sphere.

(b) Try to repeat the above calculation *without* knowing explicitly what β is.
You should actually compute the integral if possible. What coordinates should you use?

(c) The standard orientation on the unit sphere is $\omega = \sin\theta\, d\theta \wedge d\phi$. Determine $\int_{\mathbb{S}^2} \omega$.

(d) It is easy to see that $\omega = d(-\cos\theta\, d\phi)$. Doesn't part (b) imply that $\int_{\mathbb{S}^2} \omega = 0$? Explain.