MTH 434 **HW #7** Winter 2004

1. **INTEGRATION ON THE SPHERE**

(a) Choose a particular 1-form β in $\mathbb{R}^3$. Compute $\alpha = d\beta$. Show that

$$\int_{\mathbb{S}^2} \alpha = 0$$

where $\mathbb{S}^2$ denotes the unit sphere.

(b) Try to repeat the above calculation *without* knowing explicitly what β is.
You should actually compute the integral if possible. What coordinates should you use?

(c) The standard orientation on the unit sphere is $\omega = \sin\theta \, d\theta \wedge d\phi$. Determine $\int_{\mathbb{S}^2} \omega$.

(d) It is easy to see that $\omega = d(-\cos\theta \, d\phi)$. Doesn't part (b) imply that $\int_{\mathbb{S}^2} \omega = 0$? Explain.