MTH 434 **HW #5** Winter 2004

1. **SPHERICAL COORDINATES, II**

The orthonormal basis $\{\vec{\boldsymbol{e}}_1, \vec{\boldsymbol{e}}_2, \vec{\boldsymbol{e}}_3\}$ of (ordinary) vectors adapted to spherical coordinates is $\{\hat{\boldsymbol{r}}, \hat{\boldsymbol{\theta}}, \hat{\boldsymbol{\phi}}\}$, which is related to the orthonormal basis of 1-forms via

$$d\vec{\boldsymbol{r}} = dr\,\hat{\boldsymbol{r}} + r\,d\theta\,\hat{\boldsymbol{\theta}} + r\sin\theta\,d\phi\,\hat{\boldsymbol{\phi}}$$

Both sides of this equation are ***vector valued 1-forms***.

(a) Determine the exterior derivative of each basis vector (not 1-form) above, that is, compute $d\hat{\boldsymbol{r}}$, $d\hat{\boldsymbol{\theta}}$, and $d\hat{\boldsymbol{\phi}}$. *What sort of a beast should you get?*

You may calculate in any coordinate system, but the final answer should be entirely in terms of spherical coordinates and basis vectors. If you are having trouble getting started, see me.

(b) Compute $\omega_{ij} = \vec{\boldsymbol{e}}_i \cdot d\vec{\boldsymbol{e}}_j$ for $i, j = 1, 2, 3$. *What sort of a beast should you get?*

(c) Compute $R_{ij} = d\omega_{ij} + \omega_{ik} \wedge \omega_{kj}$ for $i, j = 1, 2, 3$ (and where there is an implicit sum over k).