MTH 434 **HW #2** Winter 2004

1. **INNER PRODUCTS**

Consider the function $Q(x, y, z) = xy + yz + xz$. This determines an inner product as follows. Let p be the position vector from the origin in $\mathbb{R}^3$, so that in rectangular coordinates we can write $p \doteq \begin{pmatrix} x \\ y \\ z \end{pmatrix}$. Q can then be interpreted as a symmetric matrix, also called Q, since

$$Q(x, y, z) \doteq p^T Q\, p$$

where the superscript T denotes matrix transposition. (The optional symbol $\doteq$ is used for relations which are only true in a particular basis. An ordinary equals sign is also acceptable.)

(a) Determine the matrix Q, which should be symmetric ($Q = Q^T$).

(b) Find an orthonormal basis of $\mathbb{R}^3$ under the inner product $Q(v, w) = v^T Q\, w$.

(c) Find the signature of this inner product.

(d) Find at least one null vector under this inner product (or argue that there aren't any).

(e) Rewrite the original function Q as a sum and/or difference of squares.

There are 3 different uses of the symbol Q here: the original function $Q(x, y, z)$, the matrix Q, and the inner product $Q(v, w)$. One often writes simply (v, w) for the inner product, but this makes it hard to keep track in situations where there is more than one such product.