MTH 420 – HW #3

Due on Monday, 8 February 1999

1. **ORTHOGONAL COORDINATES**

Pick any orthogonal coordinate system in (Euclidean) $\mathbb{R}^3$ other than Cartesian, cylindrical, or spherical coordinates. (You may see me for suggestions.) Working in an orthonormal basis, compute the gradient and Laplacian of an arbitrary function, and the curl and divergence of an arbitrary "vector field" (again, really a 1-form), using the expressions:

$$\begin{aligned} \nabla f &= df \\ \nabla \times \alpha &= *d\alpha \\ \nabla \cdot \alpha &= *d*\alpha \\ \triangle f = \nabla \cdot \nabla f &= *d*df \end{aligned}$$

You may check your answer in standard reference books (or with me), but you should use exterior differentiation and Hodge duality in your computation. Note that the last formula is a special case of the one preceding it.