MTH 420 – HW #2

Due on Wednesday, 3 February 1999

1. **HODGE DUAL IN MINKOWSKI SPACE**

4-dimensional Minkowski space has an orthonormal, oriented basis of 1-forms given by

$$\{dx, dy, dz, dt\}$$

i.e. $g(dx, dx) = g(dy, dy) = g(dz, dz) = 1$, $g(dt, dt) = -1$, all others zero; "volume element" (choice of orientation) given by $\omega = dx \wedge dy \wedge dz \wedge dt$.

(a) Determine the Hodge dual operator $*$ on all forms by computing its action on basis forms at each rank.

(b) How does your answer change if the opposite orientation is chosen, namely $\omega = dt \wedge dx \wedge dy \wedge dz$?

2. **SPHERICAL COORDINATES**

Consider spherical coordinates in 3-dimensional Euclidean space with the usual orientation, namely $\omega = r^2 \sin\theta \, dr \wedge d\theta \wedge d\phi$.

(a) Determine the Hodge dual operator $*$ on all forms (expressed in spherical coordinates) by computing its action on basis forms at each rank.

(b) Compute the dot and cross products of 2 arbitrary "vector fields" (really 1-forms) in spherical coordinates using the expressions:

$$\alpha \cdot \beta = *(\alpha \wedge *\beta)$$
$$\alpha \times \beta = *(\alpha \wedge \beta)$$

You may express your results either with respect to an orthonormal basis or with respect to a "coordinate" (non-orthonormal) spherical basis; make sure you know which you're doing. You can check your answer in many places, such as physics textbooks on electrodynamics, but be warned that these are almost always with respect to an orthonormal basis, and that this is not always stated explicitly!

MTH 420 – HW #3

Due on Monday, 8 February 1999

1. **ORTHOGONAL COORDINATES**

Pick any orthogonal coordinate system in (Euclidean) $\mathbb{R}^3$ other than Cartesian, cylindrical, or spherical coordinates. (You may see me for suggestions.) Working in an orthonormal basis, compute the gradient and Laplacian of an arbitrary function, and the curl and divergence of an arbitrary "vector field" (again, really a 1-form), using the expressions:

$$\nabla f = df$$
$$\nabla \times \alpha = *d\alpha$$
$$\nabla \cdot \alpha = *d{*}\alpha$$
$$\triangle f = \nabla \cdot \nabla f = *d{*}df$$

You may check your answer in standard reference books (or with me), but you should use exterior differentiation and Hodge duality in your computation. Note that the last formula is a special case of the one preceding it.