MTH 420 – HW #1

Due on Friday, 15 January 1999

DECOMPOSABLE FORMS

A p-vector $\beta \in \bigwedge^p V$ is called *decomposable* if and only if there exist vectors $\alpha^i \in V$ with

$$\beta = \alpha^1 \wedge \ldots \wedge \alpha^p.$$

An example of an *indecomposable* 2-vector is $\alpha \wedge \beta + \gamma \wedge \delta$ where $\alpha, \beta, \gamma, \delta \in V$ are linearly independent (so that $\dim V \geq 4$).

1. Let $\vec{\boldsymbol{u}}$ be a vector in $\mathbb{R}^3$, so that

$$\vec{\boldsymbol{u}} = A\,\vec{\boldsymbol{\imath}} + B\,\vec{\boldsymbol{\jmath}} + C\,\vec{\boldsymbol{k}}$$

Find 2 vectors $\vec{\boldsymbol{v}}$ and $\vec{\boldsymbol{w}}$ such that

$$\vec{\boldsymbol{u}} = \vec{\boldsymbol{v}} \times \vec{\boldsymbol{w}}$$

HINT: What properties should $\vec{\boldsymbol{v}}$ and $\vec{\boldsymbol{w}}$ have?
Start by setting one coefficient to 0, then try the general case.
Don't forget that all squares are 0!

2. Show that if $\dim V = 3$ then all 2-vectors are decomposable, in other words, show that if $\{\rho, \sigma, \tau\}$ is a basis of V then

$$\gamma = A\,\sigma \wedge \tau + B\,\tau \wedge \rho + C\,\rho \wedge \sigma$$

is decomposable.

HINT: Adapt your answer to the previous problem!
Start by setting one coefficient to 0, then try the general case.
Don't forget that all squares are 0!
(You can also solve this problem by brute force, but it's not very illuminating.)

3. Show that if $\dim V = 4$ then all 3-vectors are decomposable.

EXTRA CREDIT: Discuss the case of $(n-1)$-vectors in dimension n.
(This does not require an explicit proof.)