

SAMPLE HOMEWORK SOLUTION

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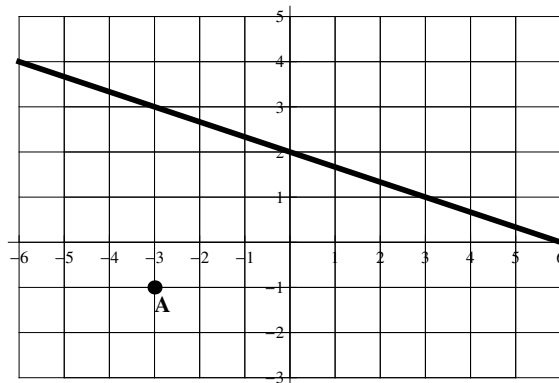


Figure 1: A map of Ideal City. Alice works at A , and the heavy line shows the mass-transit route L .

Alice works as an acrobat at $A = (-3, -1)$, but Bruno is now working as a conductor on the new mass-transit vehicle which runs along the line L shown in Figure 1. When Bruno comes to work he can get on the vehicle at the point nearest his home.

Alice and Bruno are looking for an apartment so that the distance Alice has to walk to work plus the distance Bruno has to walk to work is a minimum. Where should they look?

The shortest distance they can walk is equal to the distance between Alice's job, A , and the mass-transit line, L . The shortest path from A to L is the vertical line segment shown in Figure 2, so Alice and Bruno can clearly live anywhere along this path. But can they live anywhere else?

Moving to the left from this line segment clearly increases the distances that both Alice and Bruno would need to walk, as does moving vertically beyond the endpoints of the line segment. Moving to the right of this line segment decreases the distance that Bruno would need to walk, but also increases the distance Alice would need to walk. Since the slope m of L is

$$m = -\frac{1}{3}, \tag{1}$$

Alice must walk three blocks further for every block Bruno saves.

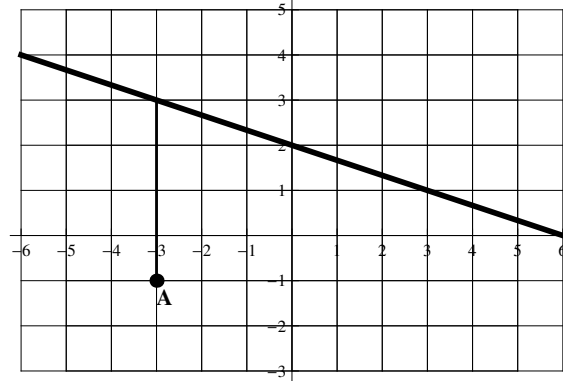


Figure 2: The vertical line shows where Alice and Bruno should live to minimize the distance they walk to work.

Therefore Alice and Bruno should live along the vertical line segment shown in Figure 2 in order to minimize the sum of the distances they both walk to work.

Unable to find an affordable apartment that minimizes the distance they walk to work, Alice and Bruno decide instead that they both want to be within six blocks of their job. Where should they look?

Alice must clearly live inside the taxicab circle of radius six centered at A , as shown in Figure 3. But what about Bruno? He must live within six blocks of L . This region can be constructed as the union of all taxicab circles centered at points on L , and consists of the points between two parallel lines six blocks “up” or “down” from L . One of these lines is shown in Figure 3.

If Alice and Bruno both want to be within six blocks of their jobs, they can live anywhere in the shaded region shown in Figure 3, which is the intersection of the two regions described above.

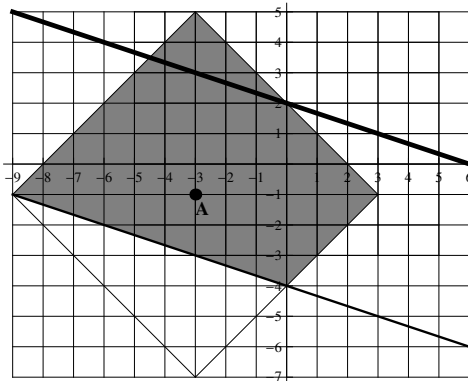


Figure 3: The shaded region shows where Alice and Bruno should live so that they are each within six blocks of their jobs.