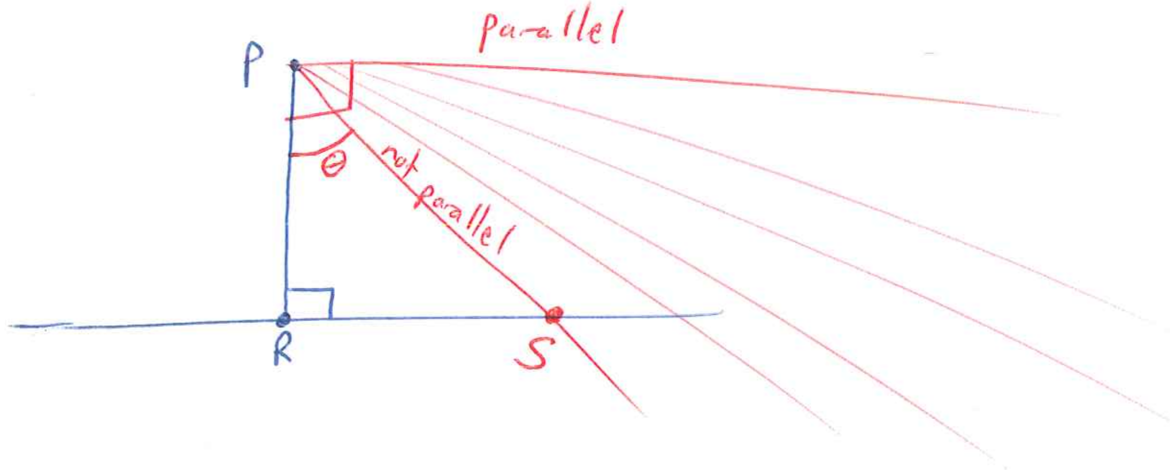
Angle of parallelism



Idea: $\exists$ an angle of parallelism θ_0 for the point P and the line $\overleftrightarrow{RS}$ such that:

$\theta < \theta_0 \Rightarrow$ not parallel

$\theta \geq \theta_0 \Rightarrow$ parallel

line at θ_0 intersects "at ∞"

"θ_0 is the angle to the first parallel line"

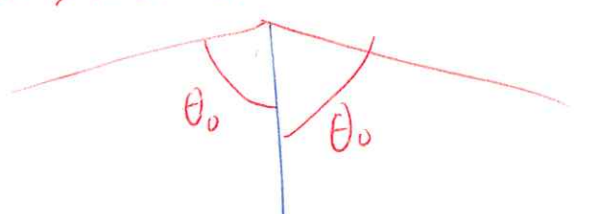
Facts:
- $0° < \theta_0 \leq 90°$

  $\theta_0 = 90° \longleftrightarrow$ Euclidean

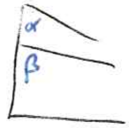
  $\theta_0 < 90° \longleftrightarrow$ hyperbolic

- θ_0 same on both sides

  (corollary to 6-2:①!)



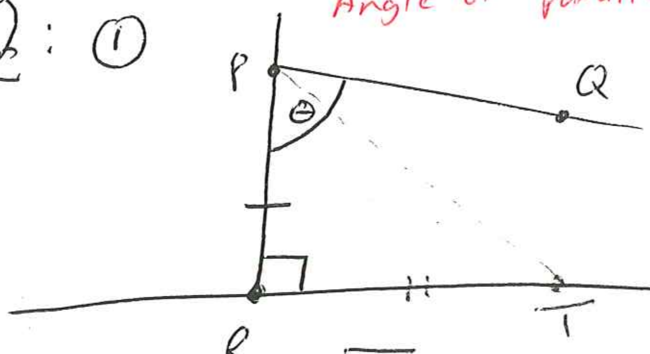
6-2:②

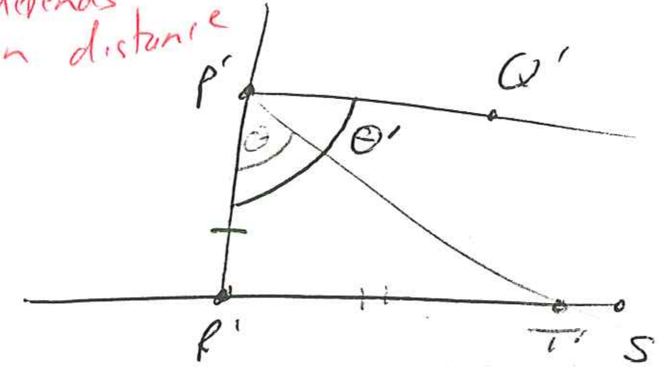
- 

$\alpha < \beta$

6-2: ①

Angle of parallelism depends only on distance





Given: $\overline{RP} = \overline{R'P'}$

Assume $\theta' > \theta$. Construct θ at P'.

By def of angle of parallelism, ray must intersect $\overleftrightarrow{R'S'}$ in a pt T'.

Now construct T on $\overleftrightarrow{RS}$ with $\overline{RT} = \overline{R'T'}$

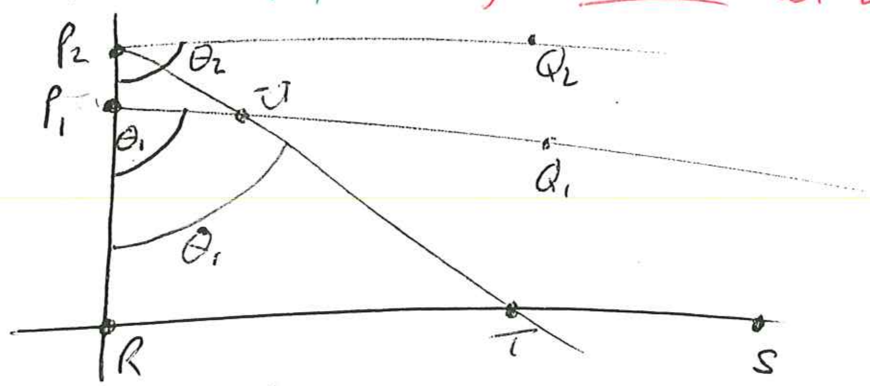
$\Longrightarrow \Delta PRT = \Delta P'R'T' \Longrightarrow \angle TPR = \theta$ $\boxed{*}$

cf Pf of Thm 6.2.2!

6-2: ②

(alternate pf on next page)

angle decreases with distance



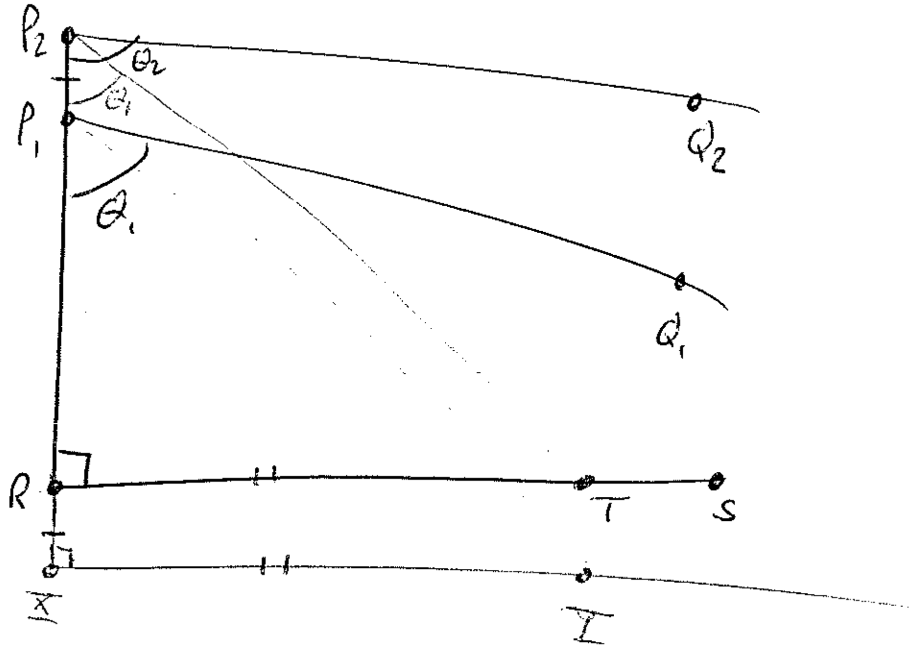
Assume $\theta_2 > \theta_1$. Construct θ_1 at P_2.

By def of angle of parallelism, ray must intersect $\overleftrightarrow{RS}$ in a pt T, & hence must cross $\overleftrightarrow{P_1Q_1}$ in a pt U.

$\Longrightarrow \Delta P_2P_1U$ has an interior and an exterior angle both $= \theta_1$ $\boxed{*}$ ("Step 2" in class cf p. 70) [exterior ∠ thm]

$\Longrightarrow \theta_2 \leq \theta_1$

6-2: 2



Assume $\theta_2 > \theta_1$

Construct θ_1 at $P_2 \Rightarrow$ resulting ray must intersect $\overleftrightarrow{RS}$ at some pt T

Extend $\overrightarrow{P_2R}$ to X such that $\overline{P_1P_2} = \overline{RX} \Rightarrow \overline{P_2R} = \overline{P_1X}$

Construct $\overline{XY} \perp \overline{P_2X}$ such that $\overline{RT} = \overline{XY}$

$\Rightarrow \triangle P_2RT = \triangle P_1XY$ by SAS

$\Rightarrow \angle RP_1Y = \angle RP_2T = \theta_1 = \angle RP_1Q_1$

$\Rightarrow \overrightarrow{P_1Q_1} = \overrightarrow{P_1Y}$ & hence must intersect $\overleftrightarrow{RS}$

$\boxed{*}$

$\therefore \theta_2 \le \theta_1$