MTH 338 **LAB 3** Winter 2022

Exploring Spherical Area

1. GETTING STARTED

This activity can be done using any model of elliptic geometry. A Lénárt Sphere is ideal, but any roughly spherical object you can write on will do fine.

If you don't have access to a sphere you can write on, you can use GeoGebra. Use either the spherical geometry applet or the Klein disk applet so that you can draw elliptic lines; both are available on the course website; direct URLs are given below. When choosing between these two applets, it may be relevant that the Pen tool only works in two-dimensional windows

The Klein disk applet is normally used to model *single* elliptic geometry, by working exclusively inside the disk, using wraparound. It can however also be used to model *spherical* geometry, by carefully considering points inside the disk, representing the Northern Hemisphere *and* points outside the disk, representing the Southern Hemisphere. Although the instructions below are written for spherical geometry, they can also be used in single elliptic geometry – with minor modifications that should be apparent as you proceed.

2. SPHERICAL GEOMETRY

- Choose a point on the sphere. Construct its antipodal point. Connect your two points with two (non-collinear) line segments.
  If using GeoGebra, you can find a pair of antipodal points by intersecting two elliptic lines. (If working in single elliptic geometry, this intersection will be a single point – preferably in the interior.)

  This shape – the lines and the region between them – is called a ***lune***.
  The ***angle*** of a lune is the angle between the two line segments, normally assumed to be less than π.
- What is the area of a sphere of radius r?
- What is the area of a lune with angle α?

3. TRIANGLES

- Construct a triangle, each of whose angles is less than π.
- From each vertex, extend the sides of the triangle to make a lune.
- Extend the sides of each lune to lines rather than line segments, thus also constructing another lune on the other side of the sphere (but not in single elliptic geometry).
  You may wish to distinguish the lunes, perhaps using different colors, if available. The Pen tool can be used in the Klein disk applet to roughly fill in different regions – the color can be changed afterward. If only a single color is available, you can use different shading, such as lines in different directions.
- How many lunes do you have? What is their combined area?
- How much of the sphere do your lunes cover?
- Derive a formula for the area of your triangle in terms of its angles.

Here are the URLs for the GeoGebra applets:

`https://math.oregonstate.edu/~tevian/onid/MTH338/handouts/spherical.html`
`https://math.oregonstate.edu/~tevian/onid/MTH338/handouts/Klein.html`