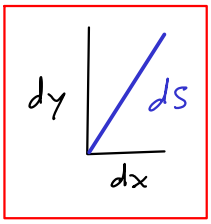
Line Elements

Plane: $ds^2 = dx^2 + dy^2$



Sphere: $ds^2 = r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$

Minkowski: $ds^2 = dx^2 - c^2 dt^2$

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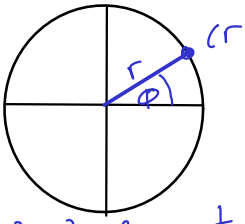
	s=0	s=1
flat	Euclidean	Minkowskian (SR)
curved	Riemannian	Lorentzian (GR)

Circle Geometry

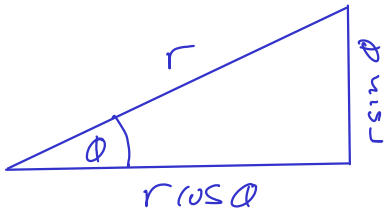


$r^2 = x^2 + y^2 = \text{const}$

- Draw circle of radius r
- Measure arclength s
- Define $\Phi = s/r$
- Define trig fns as coords

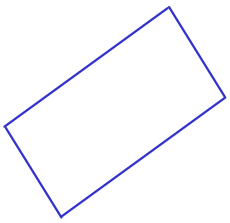
← Use line element!

Triangle Trig



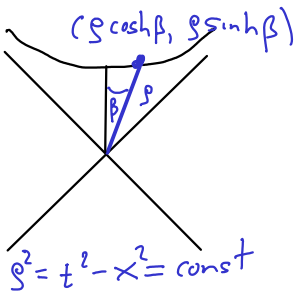
swbd: Find $\cos\theta$ if $\tan\theta = \frac{3}{4}$

Projections



How wide is the suitcase?

Hyperbola Geometry



- Draw hyperbola of radius ρ
- Measure arclength s ← Use line element!
- Define $\beta = s/\rho$
- Define trig fns as coords

Properties:

$$\cosh^2\beta - \sinh^2\beta = 1$$

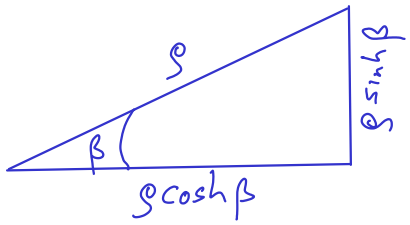
$$\cosh\beta = \frac{e^{\beta}+e^{-\beta}}{2}$$

$$\sinh\beta = \frac{e^{\beta}-e^{-\beta}}{2}$$

$$|\cosh\beta| \geq 1$$

$$|\tanh\beta| < 1$$

Triangle Trig



SWBQ: Find $\cosh\beta$ if $\tanh\beta = \frac{3}{5}$