Parallel lines exist (in neutral geometry)

Step 1: 2 distinct lines intersect in at most 1 pt

p. 23 → 29 Incidence Thm 1

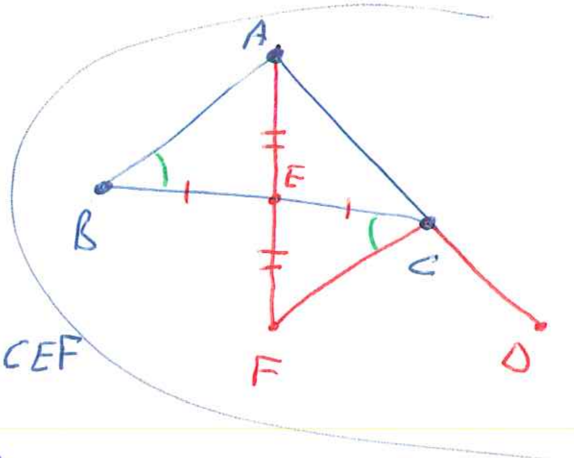
Pf: If $\exists$ 2 distinct intersection pts, then those 2 pts do not determine a unique line, which violates SMSG postulate 1 $\boxed{*}$

Step 2: The exterior angle of a triangle is bigger than either nonadjacent interior angle

p. 70 → 88 Thm 3.2.9

Pf: Given ΔABC, extend $\overrightarrow{AC}$ to some pt D. Let E be the midpoint of $\overline{BC}$, & extend $\overrightarrow{AE}$ to F such that $\overline{AE} = \overline{EF}$

uses SMSG 3 & 4



now SAS (SMSG 15) $\Rightarrow \Delta BEA = \Delta CEF$ so that $\angle ABE = \angle FCE$

uses SMSG 11 & 14

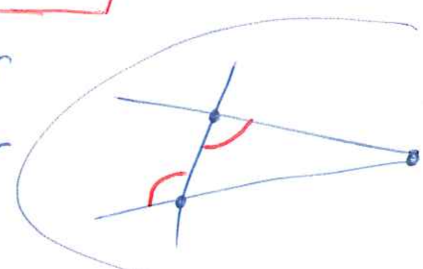
But F is in the interior of $\angle BCD$ so that $\angle DCE > \angle FCE$ ✓

uses SMSG 9 & 13

Step 3: IF 2 lines are crossed by a 3rd such that a pair of alternate interior angles are equal THEN the 2 lines are parallel

p. 76 → 95 Thm 3.4.1

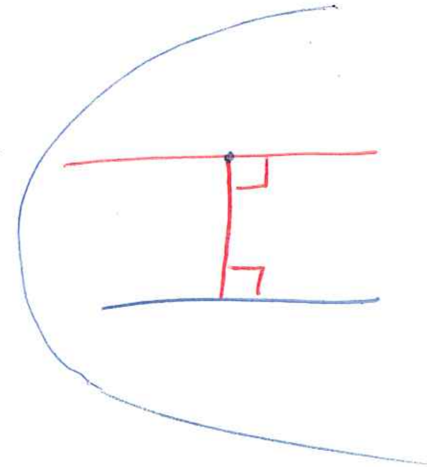
Pf: If the 2 lines intersect, then one of the angles is interior & one exterior to the resulting triangle. This violates Step 2 $\boxed{*}$



WARNING: Converse of Step 3 may be false

Step 4 : Given any line & any point not on the line $\exists$ at least one line through the point parallel to the original line

Pf : Construct line from pt $\perp$ to original line, then line through pt $\perp$ to this perpendicular.



works for any angle?

Euclidean parallel postulate :

$\exists$ 1 such parallel line

hyperbolic parallel postulate :

$\exists$ many such parallel lines

elliptic parallel postulate :

$\nexists$ parallel lines

requires changing neutral geometry!