

Spherical geometry (double elliptic geometry)

- ① points = points on unit sphere
- ② lines = ^(arcs of) n great circles on unit sphere

⇒ ① ^(distinct) all n lines intersect in 2 pts
 (replaces parallel postulate, SMSG 16)

② 2 points which are not antipodal determine
a unique great circle & hence
2 ~~lines~~ ^{segments} (replaces SMSG 1) (incidence)

③ All lines are boundless and finite
(replaces SMSG 3 - ruler)
real line \mapsto real line mod 2π
"ruler mod 2π "

plane separation OK
distance OK

Facts: angle sum > 180
 \exists rectangles

Elliptic Geometry

Want:

\nexists parallel lines

But neutral geometry $\Rightarrow \exists$ parallel lines

Key result was the Exterior Angle Thm:

The exterior angle of a triangle is bigger than either nonadjacent interior angle \rightarrow

PF: Given $\triangle ABC$, extend \overline{AC} to \overline{CD} (SKIP)

some point D . Let E be the midpoint of \overline{BC} & extend

\overline{AE} to F such that $\overline{AE} = \overline{EF}$

$\Rightarrow \triangle BEA = \triangle CEF$ by SAS

$\Rightarrow \angle ABE = \angle FCE$

But F is in interior of $\angle BCD$

$\Rightarrow \angle DCE > \angle FCE$

