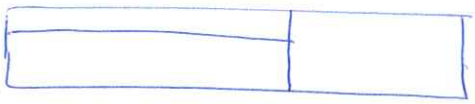
RG §3.6

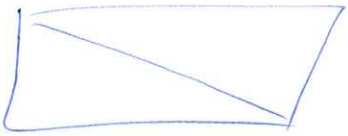
Rectangles

① $\exists 1 \Rightarrow \exists$ arbitrarily large

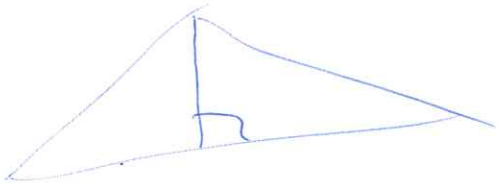
② $\exists$ arbitrarily large $\Rightarrow \exists$ any size



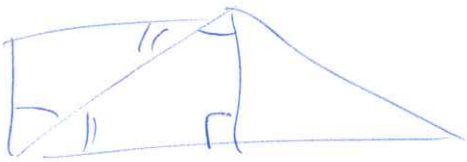
③ $\exists$ rectangle $\Rightarrow$ right triangles have angle sum 180°



④ $\Rightarrow$ all triangles have angle sum 180°

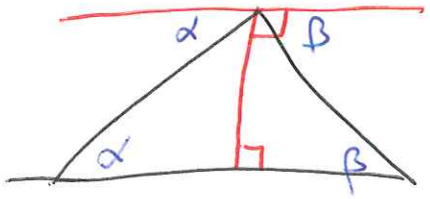


⑤ $\exists$ triangle with angle sum 180° $\Rightarrow \exists$ rectangle



$\therefore \exists$ rectangle $\Leftrightarrow \exists$ triangle with angle sum 180°
$\Leftrightarrow$ Euclidean

Euclidean $\Rightarrow$ $\angle$ sum $= 180^\circ$



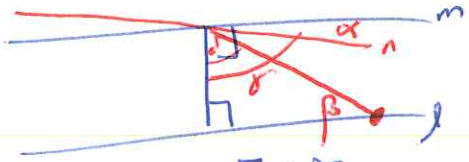
needs converse of alternate interior angle thm:

ℓ — α_1 / α_2 — m $\quad \ell \parallel m \Rightarrow \alpha_1 = \alpha_2$ (in Euclidean geometry)

Pf by contradiction: $\alpha_1 \neq \alpha_2 \Rightarrow$ can construct 2nd $\parallel$ line ⊛

$\angle$ sum $= 180^\circ \Rightarrow$ Euclidean

Pf by contradiction:



Assume $\ell \parallel m \parallel n$

- $\delta < \gamma$
- choose ℓ: $\beta < \alpha$

$\Rightarrow \beta + \delta < \alpha + \gamma = 90^\circ$ ⊛

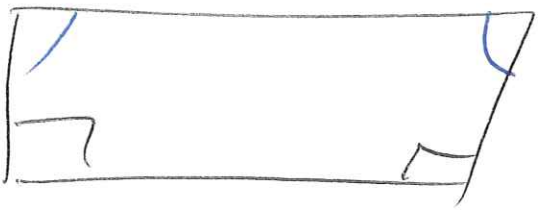
Facts

① Δ's have angle sum ≤ 180

② $\exists \Delta$ with $180 \Rightarrow$ all Δ's have 180
$\Rightarrow$ Euclidean

③ $\exists$ ▭ $\Rightarrow$ Euclidean

Saccheri



Lambert