

Euclidean geometry

"what Euclid meant" NOT "what Euclid said."

≡ SMSG (School Mathematics Study Group)
(summarized in back of book)

neutral geometry

"what Euclid meant — parallel postulate"

≡ SMSG 1st 15 postulates

Idea: use nonstandard parallel
postulate + neutral geometry

Euclid's axioms (roughly)

① 2 pts determine a unique line

② usual distance function

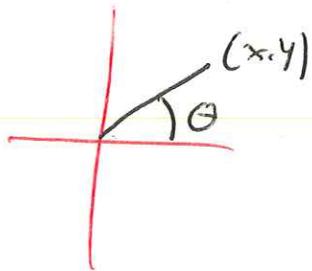
$$A = (x_1, y_1) \quad B = (x_2, y_2)$$

$$\Rightarrow d_E(A, B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

③ usual angles

e.g. angle between ray from origin to (x, y) & x -axis is

$$\tan \theta = \frac{y}{x}$$



④ parallel postulate

through a given pt not on
a given line $\exists!$ parallel line



"Review" of neutral geometry

≡ SMSG 1-15

① 2 pts determine a unique line

②-④ every line is in 1-1 correspondence with
"ruler postulates" the real numbers
(furthermore, origin & orientation are arbitrary)

⑤-⑧ & ⑩ are for 3-d

⑨ every line divides the plane into 2 disjoint sets
"plane separation" (furthermore, can't get from one side
to other w/o crossing the line)

⑪-⑭ \exists 1-1 correspondence between angles &
real numbers between 0 & 180
(for a fixed ray & a given orientation)

⑮ If 2 sides & the included angle of
"SAS" one triangle are congruent to those of another,
then the triangles are congruent.

⑯ "parallel postulate" Given a line & a point not on the line
there exists a unique line through the point
parallel to the original line

⑰-⑳ are about area & volume

Euclidean : SmSG 1-22

(but SmSG 1-16 suffice)
(SmSG 17-22 are about area & volume)

[2-d: omit SmSG 5-8 & 10]

Taxicab : SmSG 1-14 & 16

Neutral : SmSG 1-15

Hyperbolic : SmSG 1-15
+ hyperbolic parallel postulate

Elliptic / Spherical : elliptic parallel postulate
and must alter SmSG 1-15