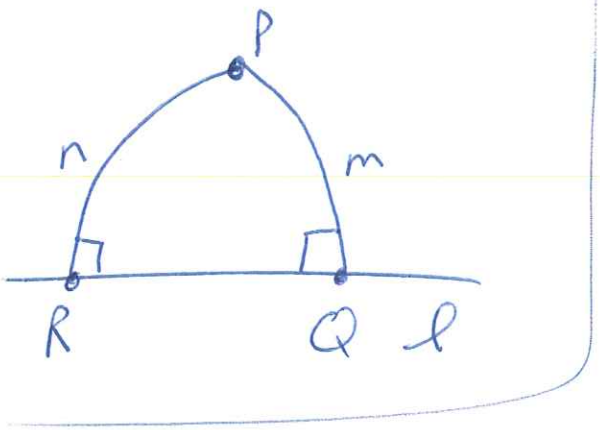
Polar Property

P ~~322~~ 405

Thm: In elliptic geometry, given any line ℓ $\exists$ at least 1 point P such that all lines from P to ℓ
① have the same length
② are $\perp$ ℓ

Pf: Pick 2 pts Q, R on ℓ.
Construct the perpendiculars m, n.
These must intersect by elliptic parallel postulate. **Claim:** P is the intersection pt



Notation: P = pole of ℓ
ℓ = polar of P
$\overline{PR}$ = polar distance

Step 1: intersection pt is same distance from ℓ along both lines

Pf: $\triangle PQR = \triangle PRQ$ by ASA

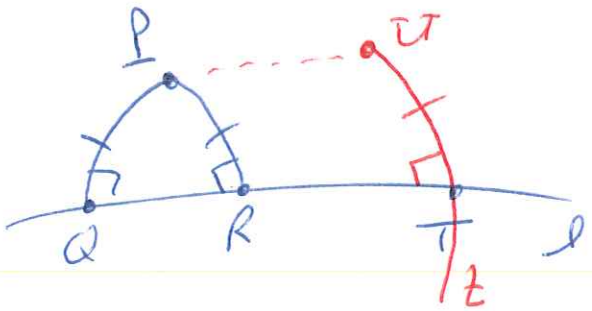
$\Rightarrow \overline{PQ} = \overline{PR}$

Step 2: $\perp$ from any pt on ℓ goes through P

Pf: Given: Q, R, T on ℓ

Construct t as above, & $\perp t$ at T

Assume P not on t



Construct U: $\overline{TU} = \overline{PR} \equiv \overline{PQ}$

$\Rightarrow \square RPUT$ & $\square QPUT$ are Saccheri quadrilaterals

$\Rightarrow$ same summit angles

$\Rightarrow \angle RPU = \angle TUP = \angle QPU$

$\Rightarrow Q = R$ ⊠

Step 3: P equidistant from Q, R, T

Pf: P on t by Step 2

$\Rightarrow P$ = intersection of m & t

$\Rightarrow \overline{PQ} = \overline{PT}$ by Step 1.

Curvature

Euclidean geometry — flat — no curvature — $S^1 \times S^1$

elliptic geometry — sphere — constant positive curvature — S^2

hyperbolic geometry — pseudosphere — constant negative curvature — all higher genus 2nd order surfaces