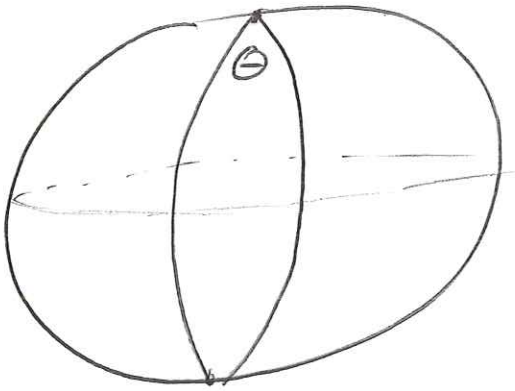
Lunes



$0 \le \theta \le 2\pi$

area of lune

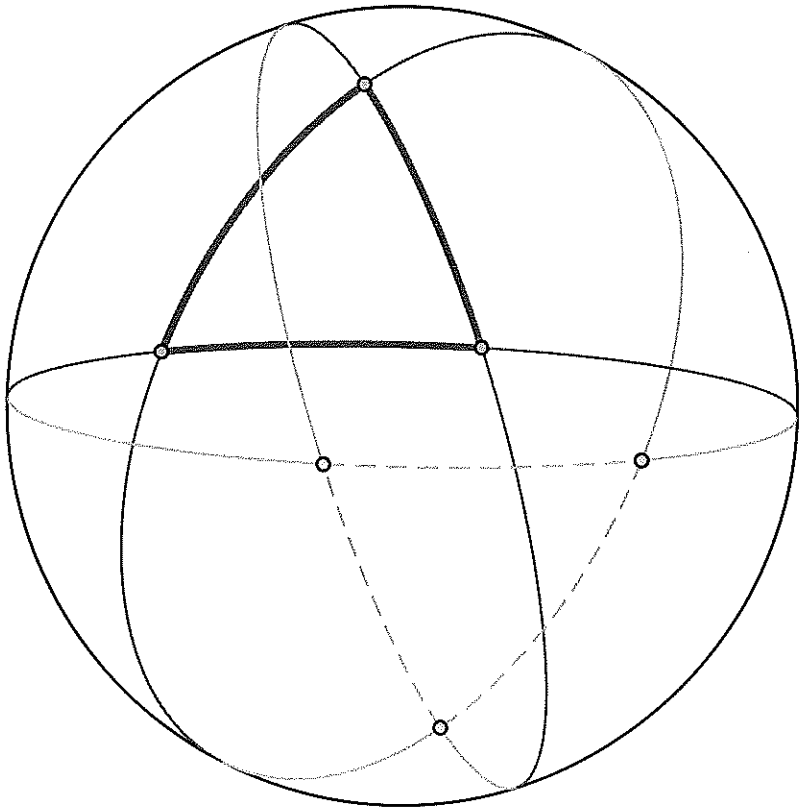
$$A(\theta) = \frac{\theta}{2\pi} \cdot 4\pi r^2$$
$$= 2\theta r^2$$

6 lunes : $A_L = 2(\alpha+\beta+\gamma) \cdot 2r^2$

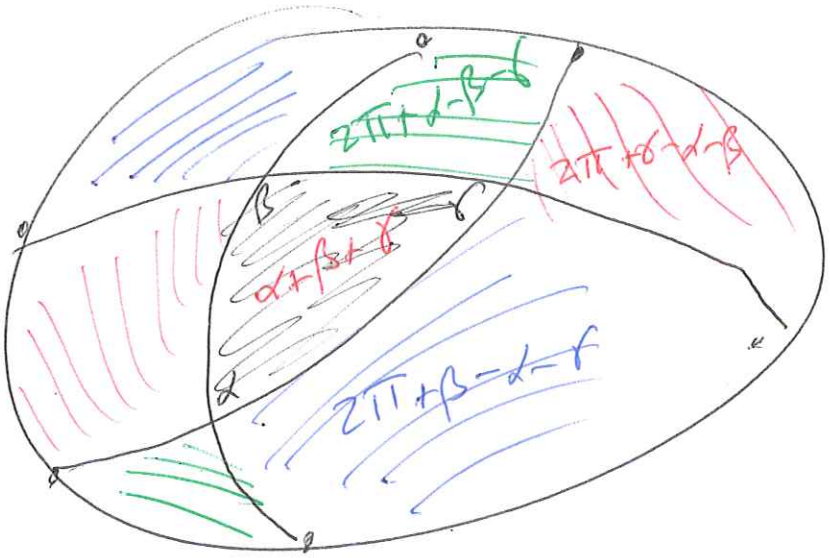
covers 2 copies of triangle 3 times each

$$\therefore A_L = 4\pi r^2 + 4A$$

$$\Rightarrow A = (\alpha+\beta+\gamma-\pi)r^2$$
$$= \varepsilon r^2$$



lunes.ps



Excess

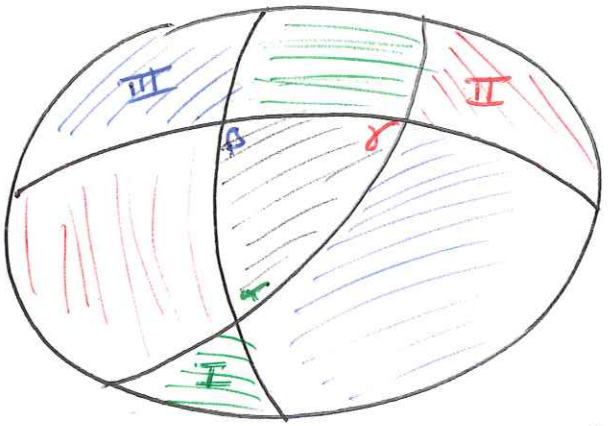
$\alpha+\beta+\gamma-\pi$

$\alpha+\beta+\pi-\gamma$

$\alpha+\gamma+\pi-\beta$

$\beta+\gamma+\pi-\alpha$

single elliptic lunes



lunes

$0 \le \Theta \le \pi$!

area of a lune

$$A(\Theta) = \frac{\Theta}{\pi} \cdot \frac{4\pi r^2}{2}$$
$$= 2\Theta r^2$$

$$\therefore A_I + A_{II} + A_{III} = 2(\alpha+\beta+\gamma)r^2$$
$$= 2\pi r^2 + 2A$$

$$\Rightarrow A = (\alpha+\beta+\gamma-\pi)r^2 = \varepsilon r^2$$

triangles

Check: $A_I = (\alpha + (\pi-\beta) + (\pi-\gamma) - \pi)r^2$
$= (\pi + \alpha - \beta - \gamma)r^2$

$A_{II} = (\pi + \gamma - \alpha - \beta)r^2$

$A_{III} = (\pi + \beta - \alpha - \gamma)r^2$

$\Rightarrow A + A_I + A_{II} + A_{III} = 2\pi r^2$ ✓