MTH 338 **LAB 3** Winter 2021

Exploring Spherical Area

1. GETTING STARTED

This activity can be done using any model of elliptic geometry. A Lénárt Sphere is ideal, but any roughly spherical object you can write on will do fine.

If you don't have access to a sphere you can write on, you can use the Klein disk GeoGebra applet to model *spherical* geometry, by carefully considering points inside the disk, representing the Northern Hemisphere *and* points outside the disk, representing the Southern Hemisphere.

You can also perform this construction in the Klein disk model of *single* elliptic geometry by working exclusively inside the disk. However, you will need to reinterpret several steps and concepts.

2. SPHERICAL GEOMETRY

- Choose a point on the sphere. Construct its antipodal point. Connect your two points with two (non-collinear) line segments.

  This shape is called a ***lune***.
  The ***angle*** of a lune is the smaller of the two angles between the two line segments.

- What is the area of a sphere of radius r?
- What is the area of a lune with angle α?

3. TRIANGLES

- Construct a triangle, each of whose angles is less than π.
- From each vertex, extend the sides of the triangle to make a lune.
- Extend the sides of each lune to lines rather than line segments, thus constructing another lune on the other side of the sphere.
- You should now have a total of 6 lunes. What is their combined area?
- How much of the sphere do your lunes cover?
- Derive a formula for the area of your triangle in terms of its angles.