

## Defects

Recall: angle sum for any hyperbolic triangle is  $< 180^\circ$  ( $S < 180$ )

Def: defect of a triangle is how much less

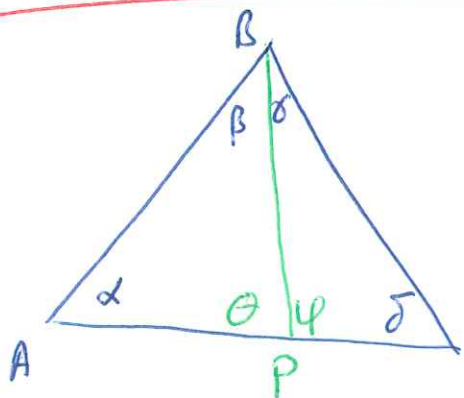
$$(D = 180 - S)$$

Excess: (elliptic)

$$E = S - 180 = -D$$

Thm: defects are additive

e.g.



$$D(\triangle ABP) = 180 - (\alpha + \beta + \theta)$$

$$D(\triangle BCP) = 180 - (\delta + \epsilon + \phi)$$

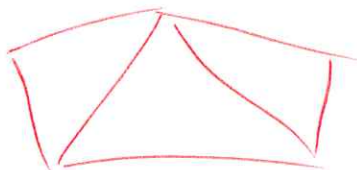
$$\frac{360 - (\alpha + \beta + \delta + \epsilon + \theta + \phi)}{180}$$

$$= 180 - (\alpha + \beta + \delta + \epsilon)$$

$$= D(\triangle ABC)$$

in particular,  $D(\triangle ABC) > D(\triangle ABP)$ , so not all defects are equal.

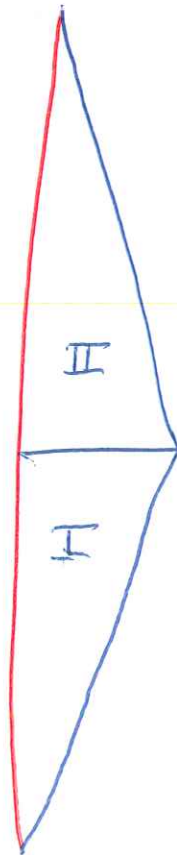
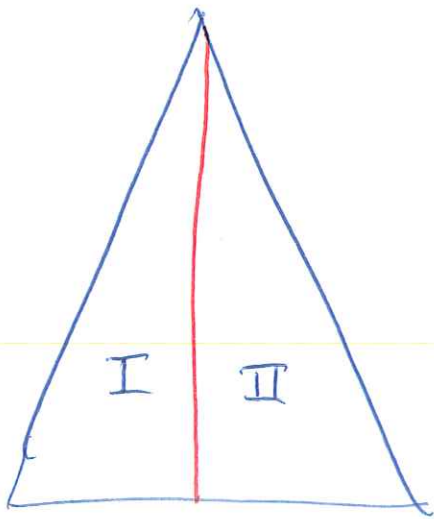
True for any (convex) polygon:  
defect is sum of defects of  
any partition into triangles



# Equivalence

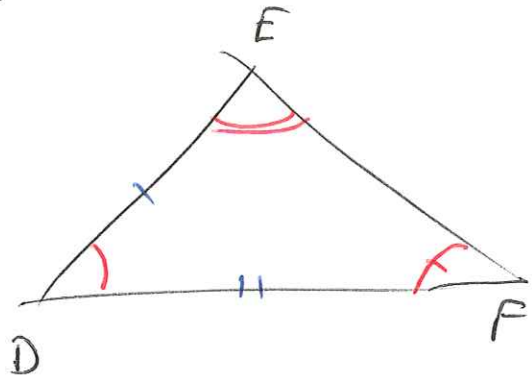
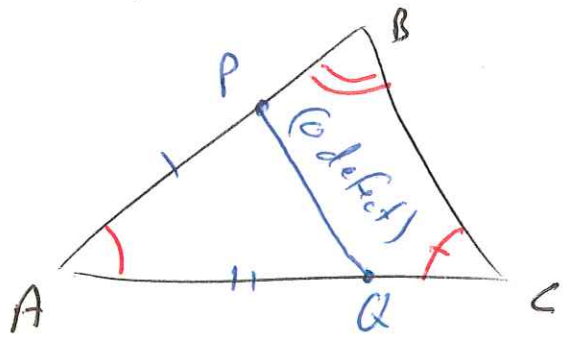
Def: Two polygons are equivalent  
 $(\Leftrightarrow)$  they can be partitioned into  
congruent sets of triangles

Idea: equivalent  $(\Leftrightarrow)$  same "area"



Thm: (AAA) In hyperbolic geometry, 2 triangles are congruent if their angles are = (no similar triangles!)

Pf: Suppose not.  $\Rightarrow$  no corresponding sides = (else ASA)  
 $\therefore$  suppose following picture:



with  $AB > DE$  &  $AC > DF$ . Construct

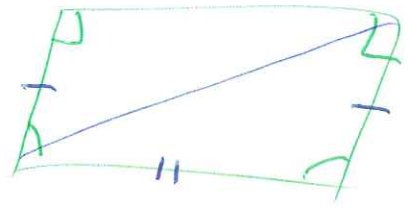
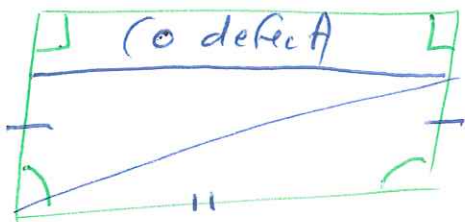
$P$  &  $Q$  :  $AP = DE$  &  $AQ = DF$

$$\Rightarrow \triangle APQ = \triangle DEF$$

$$\Rightarrow D(\triangle APQ) = D(\triangle DEF) = D(\triangle ABC)$$

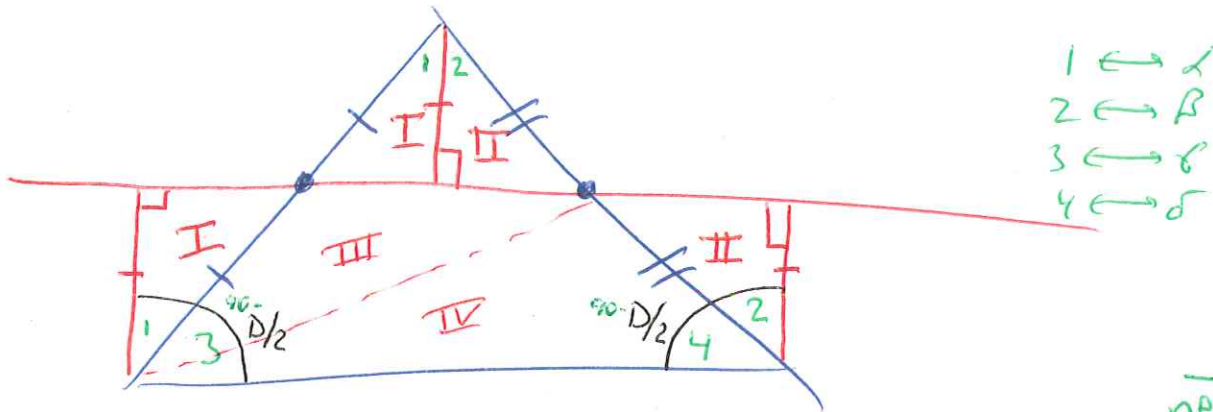
But  $D(\triangle APQ) < D(\triangle ABC)$   $\square$

Similarly, Saccheri quadrilaterals congruent if same angles + "summit"; (Pf ~~wrong~~ <sup>diff. in book? p. 277/8</sup> (upside down))



# More Interesting Example

The b.y.b. - 1  
Pf ~~was~~ complete



~~need AAS~~  
Yes!

$$D(\Delta) = 180 - (\alpha + \beta + \delta + \epsilon) =: D$$

$$D(\square) = 360 - (90 + 90 + \alpha + \delta + \beta + \epsilon) =: D$$

But summit angles of Saccheri  $\square =$

$$\Rightarrow \alpha + \delta = 90 - D/2 = \beta + \epsilon$$

(depends only on defect!)

$\Rightarrow$  Thm: Two triangles with same defect & 1 pair of congruent sides are equivalent

Pf: Corresponding Saccheri quadrilaterals have same angles & summit!

