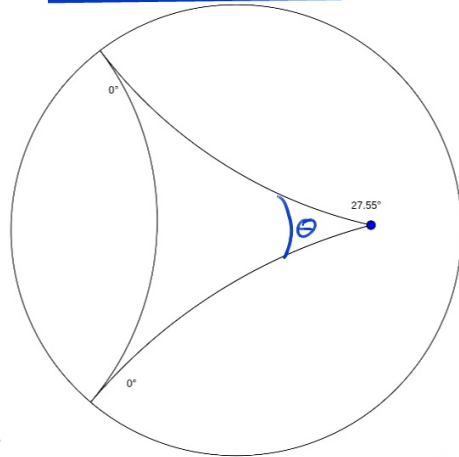


Hyperbolic Lune

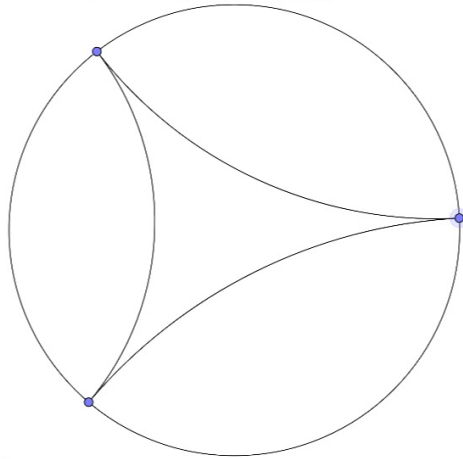


$A(\theta)$ ↓

Area depends only on θ !

why?

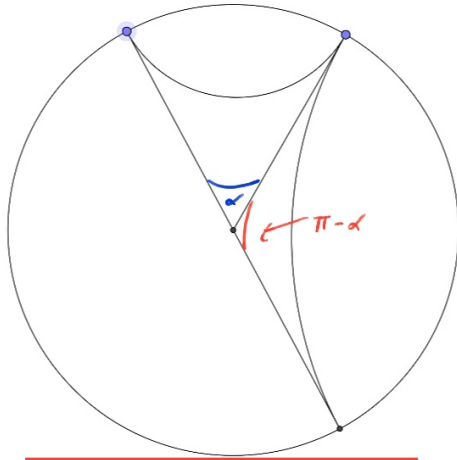
Ideal Triangle



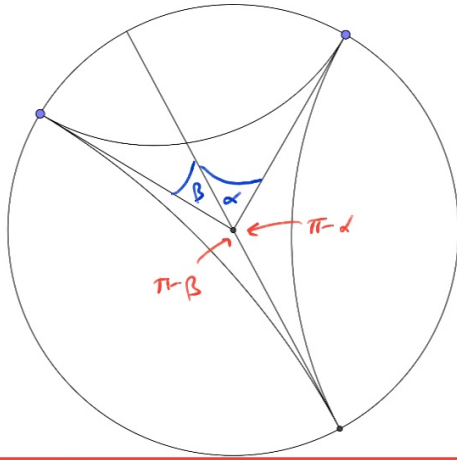
AI →

Area = constant

why?



$$A(\alpha) + A(\pi - \alpha) = A_{\text{I}}$$



$$A(\pi-d) + A(\pi-\beta) + A(d+\beta) = A_I$$

$$\Rightarrow A(\pi-d) + A(\pi-\beta) = A(\pi-(d+\beta))$$

$\swarrow \quad \searrow$
 add α & β !

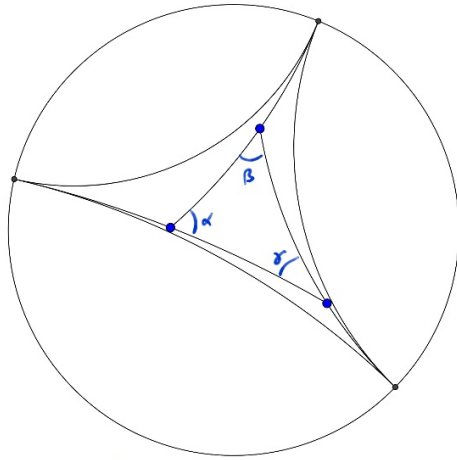
$$\therefore f(\theta) = A(\pi-\theta)$$

$$\Rightarrow f(\alpha+\beta) = f(\alpha) + f(\beta)$$

$$\Rightarrow f(\theta) = k\theta$$

$$\Rightarrow A(\pi-\theta) = k\theta$$

$$\Rightarrow A_I = A(\pi-\pi) = k\pi$$



$$\begin{aligned} \therefore A + A(\pi - \alpha) + A(\pi - \beta) + A(\pi - \gamma) &= A_I \\ \Rightarrow A &= A_I - A(\pi - (\alpha + \beta + \gamma)) \\ &= k\pi - k(\alpha + \beta + \gamma) \\ &= kD \quad \checkmark \end{aligned}$$

