

## Desargues' Theorem

### 1. GETTING STARTED

This activity is best done using software such as GeoGebra, but can also be done by hand with a straightedge.

### 2. DESARGUES' THEOREM

Desargues' Theorem states that 2 (projective) triangles are perspective with respect to a point if and only if they are perspective with respect to a line. The purpose of this lab is to construct 2 triangles which are perspective with respect to a point, and show that they are also perspective with respect to a line, thus yielding an example of (one direction of) Desargues' Theorem.

### 3. ASSIGNMENT

- Construct a triangle  $ABC$  and choose a point  $P$  outside the triangle.
- Choose points  $a$  on the line  $PA$ ,  $b$  on  $PB$ , and  $c$  on  $PC$  and construct the triangle  $abc$ .

You have constructed two triangles which are perspective with respect to  $P$ .

- Find the intersection points of corresponding sides of the triangles, that is, find the point where the line  $AB$  intersects the line  $ab$ , etc.
- The 3 intersection points should lie on a line. Do they? How can you tell?

Your 2 triangles are perspective with respect to this line.

You may have to move some of your initial points in order to fit the entire diagram on the screen, including the intersection points.

### 4. FOOD FOR THOUGHT

What does it mean if one or more of those points is at infinity? Does the result still hold?

Can you construct an example of the converse? That is, can you construct two triangles which are perspective with respect to a line, and then show that they are also perspective with respect to a point?

Does Desargues' Theorem hold in other geometries?