MTH 338 **LAB 3** Winter 2020

Exploring Spherical Area

1. GETTING STARTED

This activity can be done using any model of spherical geometry. A Lénárt Sphere is ideal, but any roughly spherical object you can write on will do in a pinch.

> You can also perform this construction in the Klein disk model of single elliptic geometry, using the tools from the previous lab. However, you will need to reinterpret several steps and concepts, starting with deciding what counts as a triangle.

2. WARMUP

- Construct a triangle with three right angles.
- (Optional) Construct some other equilateral triangle.

3. SPHERICAL GEOMETRY

- Choose a point on the sphere. Construct its antipodal point. Connect your two points with two (non-collinear) line segments.

  This shape is called a ***lune***.
  The ***angle*** of a lune is the smaller of the two angles between the two line segments.

- What is the area of a sphere of radius r?
- What is the area of a lune with angle α?

4. TRIANGLES

- Construct a triangle, each of whose angles is less than π.
- From each vertex, extend the sides of the triangle to make a lune.
- Extend the sides of each lune to lines rather than line segments, thus constructing another lune on the other side of the sphere.
- You should now have a total of 6 lunes. What is their combined area?
- How much of the sphere do your lunes cover?
- Derive a formula for the area of your triangle in terms of its angles.

5. FOOD FOR THOUGHT

If time permits, attempt one or both of the following problems.

- Construct a circle. Measure its circumference and radius. Do it again. What is π for spherical geometry?
- Can you construct an octahedron on the sphere? A cube? A tetrahedron?