3. (*cont'd.*)
   b) What do you suppose is the general shape of the ellipse, $\{P|d_E(P, A) + d_E(P, B) = 100\}$?

   c) Kepler's First Law of planetary motion states that the orbit of each planet is an ellipse with the sun at one focus. The Earth's "other" focus is about 5 million kilometers from the sun. The sum of the Earth's distances from its foci is about 300 million kilometers. What is the general shape of the Earth's orbit?

4. Again mark $A = (-2, -1)$ and $B = (2, 2)$ on a sheet of graph paper. Devise a procedure and sketch the *taxicab ellipse*

$$\{P|d_T(P, A) + d_T(P, B) = 9\}.$$

5. On a new sheet of graph paper, again mark $A = (-2, -1)$ and $B = (2, 2)$, and copy the taxicab ellipse of Exercise 4. Now sketch in different colors these other sets.

   a) $\{P|d_T(P, A) + d_T(P, B) = 13\}$

   b) $\{P|d_T(P, A) + d_T(P, B) = 7\}$

   c) $\{P|d_T(P, A) + d_T(P, B) < 13\}$

6. Sketch the taxicab ellipse, $\{P|d_T(P, M) + d_T(P, N) = 10\}$, where $M = (-2, 1)$ and $N = (4, 1)$.

7. While Euclidean ellipses have their applications in the heavens, taxicab ellipses have theirs on Earth. Alice and Bruno still have not found an apartment. Remember that Alice works at $(-3, -1)$ and Bruno works at $(3, 3)$. They have now decided that the sum of the distances that they have to walk to work should be no more than 14 blocks. Where can they look for an apartment?

8. Ajax Industrial Corporation wants to build a factory in Ideal City in a location where the sum of its distances from the rail-

Now sketch the *taxicab hyperbola*

$$\{P \mid |d_T(P, A) - d_T(P, B)| = 3\}.$$

12. On a new sheet of graph paper again mark $A = (-3, -1)$ and $B = (2, 2)$ and copy the taxicab hyperbola of Exercise 11. Using a different color for each one, sketch the following additional figures.

    a) $\{P \mid |d_T(P, A) - d_T(P, B)| = 1\}$

    b) $\{P \mid |d_T(P, A) - d_T(P, B)| = 0\}$

    c) $\{P \mid |d_T(P, A) - d_T(P, B)| = 2\}$ (Watch out!)

    d) $\{P \mid |d_T(P, A) - d_T(P, B)| = 8\}$ (Be careful!)

    e) $\{P \mid |d_T(P, A) - d_T(P, B)| = 9\}$

    What is significant about the number 8?

13. Investigate the family of taxicab hyperbolas with foci $A = (-3, 1)$ and $B = (5, 1)$.

14. Investigate the family of taxicab hyperbolas with foci $A = (0, 0)$ and $B = (4, 4)$.

15. Alice and Bruno still don't have an apartment. Their latest agreement is that neither person should have to walk more than 4 blocks farther to work than the other person. Where can they look?

8. Repeat Exercise 7 using the line $L$ through (0, 0) and (3, 1).

9. Can you think of a line $L$ for which

$$\{P|d_T(P, L) = 2\} = \{P|d_E(P, L) = 2\}?$$

10. Figure 14 shows a point $F$ and a line $L$.

    a) Sketch $\{P|d_T(P, F) = 2\}$

    b) Sketch $\{P|d_T(P, L) = 2\}$

    c) Sketch $\{P|d_T(P, F) = 2 \text{ and } d_T(P, L) = 2\}$

    d) Sketch $\{P|d_T(P, F) = d_T(P, L)\}$

11. With $F$ and $L$ as in Exercise 10, sketch

$$\{P|d_E(P, F) = d_E(P, L)\}.$$

    (A ruler and compass will be useful.)

12. The figure in Exercise 11 is known as a (Euclidean) *parabola*. $F$ is called its *focus*, $L$ its *directrix*. Thus we shall refer to

$$\{P|d_T(P, F) = d_T(P, L)\}$$

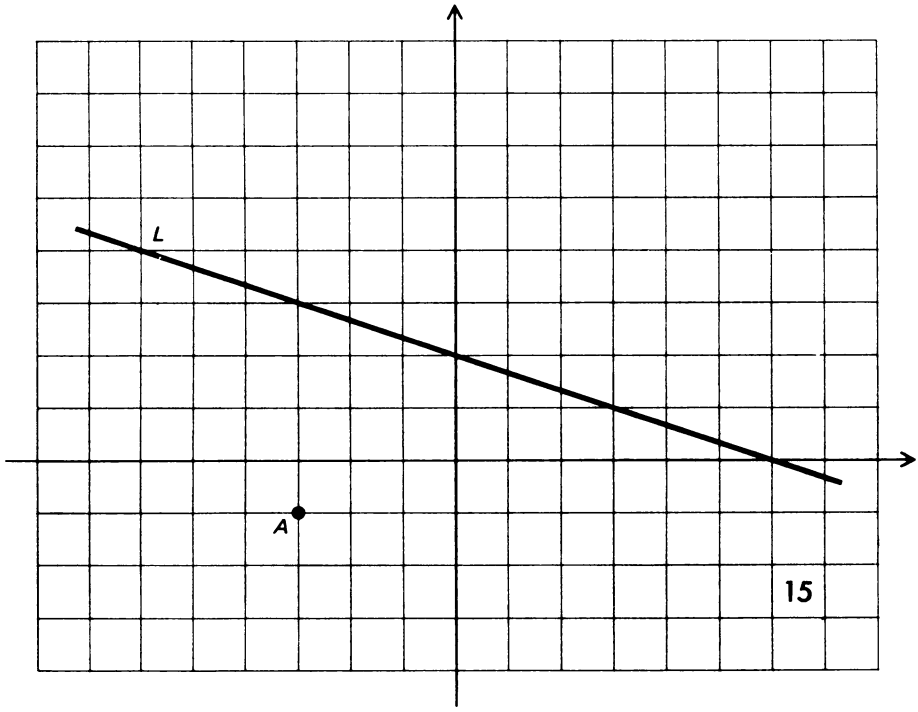
    as the *taxi parabola* with focus $F$ and directrix $L$. Sketch the taxi parabola with the focus $F$ and directrix $L$ given:

    a) $F = (-2, 2)$ $L$ is the line through $(-2, -2)$ and $(2, 2)$;

    b) $F = (0, 4)$ $L$ is the line through $(0, 0)$ and $(2, 0)$.

13. Alice still works as an acrobat at $A = (-3, -1)$, but Bruno has a new job as a conductor on the new mass-transit vehicle which runs along the line $L$ shown in Fig. 15. One of Bruno's fringe benefits is that when he comes to work he can get on the vehicle at the point nearest his home. This sends Alice and Bruno off on another apartment search.

    a) They want to live where the distance Alice has to walk to work plus the distance Bruno has to walk to work is a minimum. Where should they look?


L
A
15

13. (*cont'd.*)
    b) They change their minds and decide to live where they both have the same distance to walk to work. Where should they look?

    c) Where should they look if all that matters is that Alice have a shorter distance to walk than Bruno?

    d) Where should they look if they both want to be within 6 blocks of their job?

    e) Where should they look if the sum of the distances they have to walk is to be at most 6 blocks?

14. For old times' sake, Alice wants to walk from the amusement park $A = (-3, -1)$ to the bakery $B = (3, 3)$, but she would like to stop along the way to watch the freight train go by on the track shown in Fig. 16. Where should she stop to watch the train if she wants to minimize the distance she walks? How long will her hike from $A$ to $B$ be?

15. Repeat Exercise 14 in the case where the (straight) railroad tracks pass through $(0, -8)$ and $(4, 0)$.

16. In Fig. 17 a point $A$ and a set $S$ are shown. Approximate these two distances.

    a) $d_E(A, S)$

    b) $d_T(A, S)$