

1. Alice and Bruno are looking for an apartment in Ideal City. Alice works as an acrobat at amusement park $A = (-3, -1)$. Bruno works as a bread taster in Bakery $B = (3, 3)$. Being ecologically aware, they walk wherever they go. They have decided their apartment should be located so that the distance Alice has to walk to work plus the distance Bruno has to walk to work is as small as possible. Where should they look for an apartment?

Since they want to minimize the sum of the distances they walk, they should live somewhere “between” A and B . Intuitively, it should be possible to walk from A to B by way of their apartment without going out of their way. The possible locations for their apartment are therefore anywhere in the shaded rectangle shown in Figure 1.

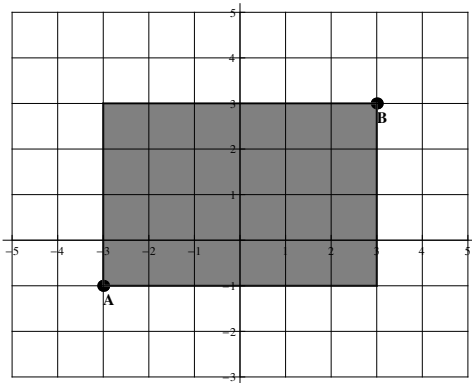


Figure 1: The possible locations for an apartment “between” A and B , solving Problem 1.

2. Alice agrees that the sum of the distances should be a minimum, but she is adamant that they both have exactly the same distance to walk to work. Now where could they live?

Since the (taxicab) distance between A and B is 10 blocks, the apartment must be located 5 blocks from both A and B . One way to determine the locations satisfying this constraint is to construct (taxicab) circles of radius 5 blocks around both A and B . The intersection of these two circles yields the desired locations, as shown by the heavy line in Figure 2.

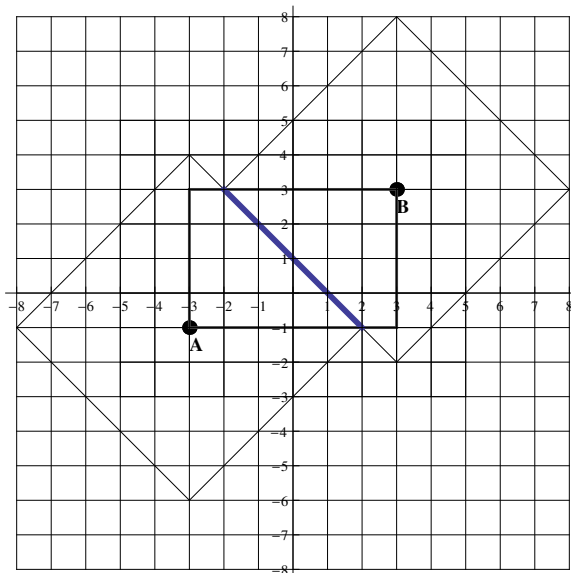


Figure 2: The intersection of circles of radius 5 blocks centered at A and B , solving Problem 2.

3. After a day of fruitless apartment hunting they decide to widen their area of search. The only requirement they keep is that they both be the same distance from their jobs. Now where should they look?

Clearly, the locations with minimum distance sum, shown in Figure 2, are still acceptable. Dropping the requirement that the distance sum be minimized, additional locations can be found by starting at the points where the line segment from Problem 2 intersects the bounding rectangle, then walking away from both points A and B . Doing so will of course increase the distance sum, but equally for Alice and Bruno. The additional locations are along the heavy vertical lines shown in Figure 3.

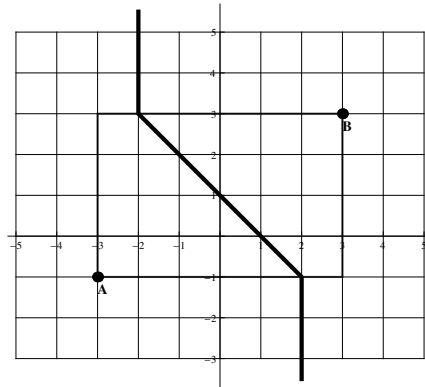


Figure 3: The *midset* of A and B (all points equidistant from these two points), solving Problem 3.