Exploring Elliptic Geometry Using Geometer's Sketchpad

1. GETTING STARTED

- Navigate to `\\poole.scf.oregonstate.edu\ClassFolders\Math–Dray\MTH338` (in a folder window, not a browser).

2. ELLIPTIC GEOMETRY

- Copy the sketch `Elliptic` to your ONID directory, then open ***the copy***.

  > The filename is `Elliptic.gsp` . Opening my copy of this file may prevent others in the class from accessing it.

  This sketch not only shows the Klein disk, but also adds custom tools to the last icon on the left, below the text icon. To access one of these tools the first time, click on the icon, then select the tool you want. To access the same tool again, after having selected another tool, simply click on the icon again. ***To get the list of tools back, click on the icon and hold the mouse button down until the list reappears.***

  > The custom tools in this sketch occasionally assign the same label to different points. You can change the labels by selecting `Label Point` from the `Display` menu (or after right-clicking on the point).

3. ASSIGNMENT

- Construct an elliptic triangle. Measure all of its sides and angles.
- Construct another triangle such that two corresponding sides as well as the included angle are congruent.
- Check whether SAS congruence holds in this case by measuring the remaining sides and angles.
- (Optional) Construct an equilateral triangle. Measure the angles. Are they the same? Should they be? What happens if you change the size of the triangle?
- Construct a (complete) circle in the Klein Disk which intersects the "equator" in exactly two (elliptic!) points. That is, find the set of all points *in the Klein Disk* that are a given distance from a given point, with the further restriction that the resulting set of points must contain exactly two (elliptic) points on the boundary.

Exploring Elliptic Geometry Using Mathematica

4. GETTING STARTED

- Navigate to `\\poole.scf.oregonstate.edu\ClassFolders\Math-Dray`
  (in a folder window, not a browser).
- Copy `MathStart.nb` to your ONID filespace and rename it `Klein.nb` .
- Double-click (your copy of) `Klein.nb` to start *Mathematica*.

5. SPHERICAL GEOMETRY

Recall that elliptic geometry comes in two flavors: spherical geometry, also known as *double elliptic geometry*, and the Klein Disk, which is a model of *single elliptic geometry*. The Klein Disk can be thought of as representing the Northern Hemisphere of a sphere, with an appropriate concept of wraparound at the equator. Without the wraparound feature, it can thus be used as a (partial) model of spherical geometry as well.

Some things to try (execute each command separately):

`<<Klein.m`	Load Tevian's elliptic geometry package.
`P={0.6,0}`	Let P be the point $(-0.6,0)$.
`Q={0,0.9}`	
`R={-0.3,-0.4}`	
`EDraw[ELine["P","Q"]]`	Draw the line through the given points.
`EDraw[ETriangle["P","Q","R"]]`	Draw the triangle through the given points.

The elliptic geometry package contains 3 distance functions, which you should now explore:

`SDistance[P,-P]`	Find the *spherical* distance.
`EDistance[P,-P]`	Find the distance *in the Klein Disk*.
`EarthDistance[Portland,SanFrancisco]`	Find the distance *on the Earth* in miles.
`EDraw[EarthLine[Portland,SanFrancisco]]`	Draw the great circle on the Earth.

Why are the first two results different? Try this for other points.

Cities are designated by their (north) latitude and (west) longitude. The predefined cities are Anchorage, Cincinnati, Corvallis, Edmonton, Honolulu, London, NewYork, Portland, SanFrancisco, and Tokyo; feel free to create your own. There are also corresponding functions `EarthPoint` and `EarthTriangle`. WARNING: Only cities in the Northern Hemisphere will wind up inside the unit circle, with the North Pole at the center. (There are also commands `EarthLineP` and `EarthTriangleP` which center the Pacific Ocean in the unit circle.)

6. ASSIGNMENT

- **Draw a diagram showing the direct route from New York to Tokyo, as well as the indirect routes via Anchorage, San Francisco, and Honolulu. Print the result, and label each city (by hand) and give the distances of each leg as well as the total distance for each routing.**

  You can print part of a notebook by selecting what you want, then using the menu item `Print Selection` .