MTH 338 **LAB 3** Spring 2012

Exploring Spherical Area

1. GETTING STARTED

This activity can be done using any model of spherical geometry. A Lénárt Sphere is ideal, but any roughly spherical object you can write on will do in a pinch.

This construction can be performed electronically using a Java applet for modeling spherical geometry, which can be found at `http://oregonstate.edu/~drayt/MTH338/java/easel.html` .

You can also perform this construction in the Klein disk model of single elliptic geometry, using the tools from the previous lab (either the `Elliptic.gsp` sketch in Geometer's Sketchpad, or the `Klein.m` package for *Mathematica*). However, you will need to reinterpret several steps and concepts, starting with deciding what counts as a triangle.

2. WARMUP

- Construct a triangle with three right angles.
- (Optional) Construct some other equilateral triangle.

3. SPHERICAL GEOMETRY

- Choose a point on the sphere. Construct its antipodal point. Connect your two points with two (non-collinear) line segments.

This shape is called a ***lune***.
The ***angle*** of a lune is the smaller of the two angles between the two line segments.

- What is the area of a sphere of radius r?
- What is the area of a lune with angle α?

4. TRIANGLES

- Construct a triangle, each of whose angles is less than π.
- From each vertex, extend the sides of the triangle to make a lune.
- Extend the sides of each lune to lines rather than line segments, thus constructing another lune on the other side of the sphere.
- You should now have a total of 6 lunes. What is their combined area?
- How much of the sphere do your lunes cover?
- Derive a formula for the area of your triangle in terms of its angles.

5. FOOD FOR THOUGHT

If time permits, attempt one or both of the following problems.

- Construct a circle. Measure its circumference and radius. Do it again. What is π for spherical geometry?
- Can you construct an octahedron on the sphere? A cube? A tetrahedron?