Exploring Elliptic Geometry Using Mathematica

1. INTRODUCTION

These notes provide a very brief introduction to the use of *Mathematica* on the computers in Kidder 108, and should also work anywhere on campus where *Mathematica* is installed.

2. GETTING STARTED

- Add the network place `\\poole\ClassFolders` .
- Browse to `\\poole\ClassFolders\Math-Dray` .
- Double-click on `Map Onid Drive` .
- Copy `MathStart.nb` to your ONID filespace and rename it `Klein.nb` .
- Double-click (your copy of) `Klein.nb` to start *Mathematica*.

To execute a command, hold down the shift key while pressing enter.

You may need to open sections in order to see the commands they contain. To do so, double-click on the bracket near the right edge of the screen which spans the section you want to open.

You need to select a command before executing it, either by clicking anywhere in the text, or by single-clicking the bracket at the right. (By default, *Mathematica* expects you to type a new command, rather than jumping to the next command.)

3. SPHERICAL GEOMETRY

Recall that elliptic geometry comes in two flavors: spherical geometry, also known as *double elliptic geometry*, and the Klein Disk, which is a model of *single elliptic geometry*. The Klein Disk can be thought of as representing the Northern Hemisphere of a sphere, with an appropriate concept of wraparound at the equator. Without the wraparound feature, it can thus be used as a (partial) model of spherical geometry as well.

Some things to try:

`<<Klein.m`	Load Tevian's elliptic geometry package.
`P={0.6,0}`	Let P be the point (−0.6,0).
`Q={0,0.9}`	
`R={-0.3,-0.4}`	
`EDraw[ELine["P","Q"]]`	Draw the line through the given points.
`EDraw[ETriangle["P","Q","R"]]`	Draw the triangle through the given points.

The elliptic geometry package contains 3 distance functions, which you should now explore:

`SDistance[P,-P]`	Find the *spherical* distance.
`EDistance[P,-P]`	Find the distance *in the Klein Disk*.
`EarthDistance[Portland,SanFrancisco]`	Find the distance *on the Earth* in miles.
`EDraw[EarthLine[Portland,SanFrancisco]]`	Draw the great circle on the Earth.

Why are the first 2 results different? Try this for other points.

Cities are designated by their (north) latitude and (west) longitude. The predefined cities are Anchorage, Cincinnati, Corvallis, Edmonton, Honolulu, London, NewYork, Portland, SanFrancisco, and Tokyo; feel free to create your own. There are also corresponding functions `EarthPoint` and `EarthTriangle`. WARNING: Only cities in the Northern Hemisphere will wind up inside the unit circle, with the North Pole at the center. (There are also commands `EarthLineP` and `EarthTriangleP` which center the Pacific Ocean in the unit circle.)

4. SAS

The commands below explore SAS congruence in the Klein Disk.

`T1:=ETriangle["A1","B1","C1"]`	
`EDraw[T1]`	
`C2={0.4,0}`	
`A2=XFindPoint[A1,C1,C2]`	Construct $A2$ so that $d(A2, C2) = d(A1, C1)$.
`B2=HFindPoint[C1,B1,C2]`	Construct $B2$ so that $d(B2, C2) = d(B1, C1)$.
`T2:=ETriangle["A2","B2","C2"]`	
`EDraw[T1,T2]`	Compare the new triangle with the old.
`EDistance[A1,B1]`	Find the (elliptic) distance from A1 to B1.
`EDistance[A2,B2]`	Compare with corresponding side above.

5. TASKS

- **Draw a diagram showing the direct route from New York to Tokyo, as well as the indirect routes via Anchorage, San Francisco, and Honolulu. Print the result, and label each city (by hand) and give the distances of each leg as well as the total distance for each routing.**

  You can print part of a notebook by selecting what you want, then using the menu item `Print Selection` .

- **Select a point $B3$ somewhere on the y-axis and a point $A3$ somewhere on the x-axis (both points should be within the unit circle), and construct the (right) triangle $T3$ with vertices $A3$, $B3$, $C3 = C1$.**
- **Select a point $C4$ somewhere on the x-axis (again, within the unit circle), and construct the points $A4$, $B4$ using the procedure which was used above to construct $A2$, $B2$.**

  The commands `XFindPoint` and `HFindPoint` should return points within the unit circle even if you start near the edge.

- **Construct the (right) triangle $T4$ with vertices $A4$, $B4$, $C4$.**
- **Draw the triangles $T3$ and $T4$ together, and print the result using `PSPrint`. Determine the lengths of all 6 sides of these 2 triangles and label your output.**