

1. Water in a bathtub has velocity vector field near the drain given by

$$\vec{F} = -\frac{cy + bxz}{(z^2 + a^2)^2} \hat{i} - \frac{byz - cx}{(z^2 + a^2)^2} \hat{j} - \frac{b}{z^2 + a^2} \hat{k}$$

with x, y, z, a in cm, b in $\frac{\text{cm}^3}{\text{sec}}$, and c in $\frac{\text{cm}^4}{\text{sec}}$, so that \vec{F} is in $\frac{\text{cm}}{\text{sec}}$.

- (a) Rewriting \vec{F} as follows, describe in words how the water is moving:

$$\vec{F} = \frac{c(-y\hat{i} + x\hat{j})}{(z^2 + a^2)^2} + \frac{-bz(x\hat{i} + y\hat{j})}{(z^2 + a^2)^2} - \frac{b}{z^2 + a^2} \hat{k}$$

(You may assume that $a, b,$ and c are all positive.)

- (b) The drain in the bathtub is a disk in the xy -plane with center at the origin and radius 1 cm. Find the rate at which the water is leaving the bathtub. (That is, find the rate at which water is flowing through the disk.) Give units for your answer.
- (c) Find the divergence of \vec{F} .
- (d) Find the flux of the water through the hemisphere of radius 1, centered at the origin, lying below the xy -plane and oriented downward.
- (e) Find $\int_C \vec{G} \cdot d\vec{r}$ where C is the edge of the drain, orientated clockwise when viewed from above, and where

$$\vec{G} = \frac{1}{2} \left(\frac{b(y\hat{i} - x\hat{j})}{z^2 + a^2} - \frac{c(x^2 + y^2)}{(z^2 + a^2)^2} \hat{k} \right)$$

- (f) Calculate the curl of \vec{G} .
- (g) Explain why your answer to parts (d) and (e) are equal.