HW #7

1. A smooth vector field $\vec{\boldsymbol{G}}$ satisfies

$$\left(\vec{\nabla} \times \vec{\boldsymbol{G}}\right)\Big|_{(0,0,0)} = 2\,\hat{\boldsymbol{\imath}} - 3\,\hat{\boldsymbol{\jmath}} + 5\,\hat{\boldsymbol{k}}$$

Estimate the circulation $\oint \vec{\boldsymbol{G}} \cdot d\vec{\boldsymbol{r}}$ around a circle of radius 0.01 centered at the origin in each of the following planes:

(a) xy-plane, oriented counterclockwise when viewed from the positive z-axis.

(b) yz-plane, oriented counterclockwise when viewed from the positive x-axis.

(c) xz-plane, oriented counterclockwise when viewed from the positive y-axis.

2. Water in a bathtub has velocity vector field near the drain given by

$$\vec{\boldsymbol{F}} = -\frac{y + xz}{(z^2+1)^2}\,\hat{\boldsymbol{\imath}} - \frac{yz - x}{(z^2+1)^2}\,\hat{\boldsymbol{\jmath}} - \frac{1}{z^2+1}\,\hat{\boldsymbol{k}}$$

with x, y, z in cm, and $\vec{\boldsymbol{F}}$ in $\frac{\text{cm}}{\text{sec}}$.

(a) Rewriting $\vec{\boldsymbol{F}}$ as follows, describe in words how the water is moving:

$$\vec{\boldsymbol{F}} = \frac{-y\,\hat{\boldsymbol{\imath}} + x\,\hat{\boldsymbol{\jmath}}}{(z^2+1)^2} + \frac{-z(x\,\hat{\boldsymbol{\imath}} + y\,\hat{\boldsymbol{\jmath}})}{(z^2+1)^2} - \frac{1}{z^2+1}\,\hat{\boldsymbol{k}}$$

(b) The drain in the bathtub is a disk in the xy-plane with center at the origin and radius 1 cm. Find the rate at which the water is leaving the bathtub. (That is, find the rate at which water is flowing through the disk.) Give units for your answer.

(c) Find the divergence of $\vec{\boldsymbol{F}}$.

(d) Find the flux of the water through the hemisphere of radius 1, centered at the origin, lying below the xy-plane and oriented downward.

(e) Find $\int_C \vec{\boldsymbol{G}} \cdot d\vec{\boldsymbol{r}}$ where C is the edge of the drain, orientated clockwise when viewed from above, and where

$$\vec{\boldsymbol{G}} = \frac{1}{2}\left(\frac{y\,\hat{\boldsymbol{\imath}} - x\,\hat{\boldsymbol{\jmath}}}{z^2+1} - \frac{x^2+y^2}{(z^2+1)^2}\,\hat{\boldsymbol{k}}\right)$$

(f) Calculate the curl of $\vec{\boldsymbol{G}}$.

(g) Explain why your answer to parts (d) and (e) are equal.