MTH 255 **HW #6** Winter 2011

1. Suppose $\vec{\nabla} \cdot \vec{\boldsymbol{F}} = xyz^2$.

(a) Find $\vec{\nabla} \cdot \vec{\boldsymbol{F}}$ at the point $(1, 2, 1)$.
*Note: You are given $\vec{\nabla} \cdot \vec{\boldsymbol{F}}$, not $\vec{\boldsymbol{F}}$!*

(b) Using your answer to part (a), but no other information about the vector field $\vec{\boldsymbol{F}}$, estimate the flux out of a small box of side 0.2 centered at the point $(1, 2, 1)$ and with edges parallel to the axes.

(c) Without computing the vector field $\vec{\boldsymbol{F}}$, calculate the exact flux out of the box.

2. A smooth vector field $\vec{\boldsymbol{G}}$ satisfies

$$\left(\vec{\nabla} \times \vec{\boldsymbol{G}}\right)\Big|_{(0,0,0)} = 2\,\hat{\boldsymbol{\imath}} - 3\,\hat{\boldsymbol{\jmath}} + 5\,\hat{\boldsymbol{k}}$$

Estimate the circulation $\oint \vec{\boldsymbol{G}} \cdot d\vec{\boldsymbol{r}}$ around a circle of radius 0.01 centered at the origin in each of the following planes:

(a) $xy$-plane, oriented counterclockwise when viewed from the positive $z$-axis.

(b) $yz$-plane, oriented counterclockwise when viewed from the positive $x$-axis.

(c) $xz$-plane, oriented counterclockwise when viewed from the positive $y$-axis.