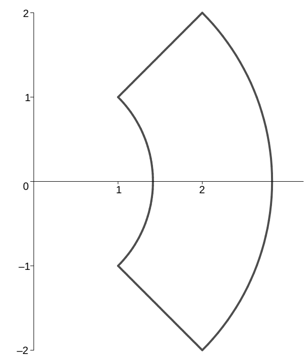
MTH 255 **HW #3** Winter 2011

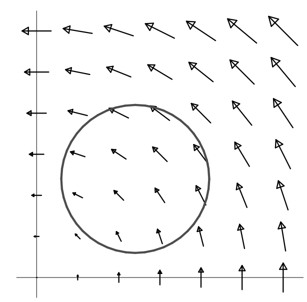


1. For each of the problems below, say whether you expect the given vector field to have positive, negative, or zero circulation *counterclockwise* around the closed curve C in the figure shown at the right. Two of the segments of C are circular arcs centered at the origin; the other two are radial line segments. You may find it helpful to sketch the vector field.

(a) $\vec{\boldsymbol{G}} = x\,\hat{\boldsymbol{\imath}} + y\,\hat{\boldsymbol{\jmath}}$

(b) $\vec{\boldsymbol{H}} = y\,\hat{\boldsymbol{\imath}} - x\,\hat{\boldsymbol{\jmath}}$

2. Consider the vector field $\vec{\boldsymbol{F}}$ shown at the right, and the loop C, which is to be traversed in the *counterclockwise* direction.



(a) Is $\displaystyle\oint_C \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{r}}$ positive, negative, or zero?

(b) From your answer to part (a), can you determine whether or not $\vec{\boldsymbol{F}} = \vec{\nabla} f$ for some function f?

(c) Which of the following formulas best fits $\vec{\boldsymbol{F}}$?

$$\vec{\boldsymbol{F}}_1 = \frac{x}{x^2+y^2}\,\hat{\boldsymbol{\imath}} + \frac{y}{x^2+y^2}\,\hat{\boldsymbol{\jmath}}$$

$$\vec{\boldsymbol{F}}_2 = -y\,\hat{\boldsymbol{\imath}} + x\,\hat{\boldsymbol{\jmath}}$$

$$\vec{\boldsymbol{F}}_3 = \frac{-y}{(x^2+y^2)^2}\,\hat{\boldsymbol{\imath}} + \frac{x}{(x^2+y^2)^2}\,\hat{\boldsymbol{\jmath}}$$

3.

(a) For each vector field $\vec{\boldsymbol{F}}$ shown below, sketch a curve for which the integral $\displaystyle\int_C \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{r}}$ is positive.

(b) For which of these vector fields is it possible to choose your curve to be closed?