VECTOR DERIVATIVES

Rectangular coordinates

$$\begin{aligned} d\vec{r} &= dx\,\hat{\boldsymbol{x}} + dy\,\hat{\boldsymbol{y}} + dz\,\hat{\boldsymbol{z}} \\ \vec{F} &= F_x\,\hat{\boldsymbol{x}} + F_y\,\hat{\boldsymbol{y}} + F_z\,\hat{\boldsymbol{z}} \end{aligned}$$

$$\vec{\boldsymbol{\nabla}}f = \frac{\partial f}{\partial x}\,\hat{\boldsymbol{x}} + \frac{\partial f}{\partial y}\,\hat{\boldsymbol{y}} + \frac{\partial f}{\partial z}\,\hat{\boldsymbol{z}}$$

$$\vec{\boldsymbol{\nabla}}\cdot\vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

$$\vec{\boldsymbol{\nabla}}\times\vec{F} = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\hat{\boldsymbol{x}} + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right)\hat{\boldsymbol{y}} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\hat{\boldsymbol{z}}$$

Cylindrical coordinates

$$\begin{aligned} d\vec{r} &= dr\,\hat{\boldsymbol{r}} + r\,d\phi\,\hat{\boldsymbol{\phi}} + dz\,\hat{\boldsymbol{z}} \\ \vec{F} &= F_r\,\hat{\boldsymbol{r}} + F_\phi\,\hat{\boldsymbol{\phi}} + F_z\,\hat{\boldsymbol{z}} \end{aligned}$$

$$\vec{\boldsymbol{\nabla}}f = \frac{\partial f}{\partial r}\,\hat{\boldsymbol{r}} + \frac{1}{r}\frac{\partial f}{\partial \phi}\,\hat{\boldsymbol{\phi}} + \frac{\partial f}{\partial z}\,\hat{\boldsymbol{z}}$$

$$\vec{\boldsymbol{\nabla}}\cdot\vec{F} = \frac{1}{r}\frac{\partial}{\partial r}\left(rF_r\right) + \frac{1}{r}\frac{\partial F_\phi}{\partial \phi} + \frac{\partial F_z}{\partial z}$$

$$\vec{\boldsymbol{\nabla}}\times\vec{F} = \left(\frac{1}{r}\frac{\partial F_z}{\partial \phi} - \frac{\partial F_\phi}{\partial z}\right)\hat{\boldsymbol{r}} + \left(\frac{\partial F_r}{\partial z} - \frac{\partial F_z}{\partial r}\right)\hat{\boldsymbol{\phi}} + \frac{1}{r}\left(\frac{\partial}{\partial r}\left(rF_\phi\right) - \frac{\partial F_r}{\partial \phi}\right)\hat{\boldsymbol{z}}$$

Spherical coordinates

$$\begin{aligned} d\vec{r} &= dr\,\hat{\boldsymbol{r}} + r\,d\theta\,\hat{\boldsymbol{\theta}} + r\,\sin\theta\,d\phi\,\hat{\boldsymbol{\phi}} \\ \vec{F} &= F_r\,\hat{\boldsymbol{r}} + F_\theta\,\hat{\boldsymbol{\theta}} + F_\phi\,\hat{\boldsymbol{\phi}} \end{aligned}$$

$$\vec{\boldsymbol{\nabla}}f = \frac{\partial f}{\partial r}\,\hat{\boldsymbol{r}} + \frac{1}{r}\frac{\partial f}{\partial \theta}\,\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\,\hat{\boldsymbol{\phi}}$$

$$\vec{\boldsymbol{\nabla}}\cdot\vec{F} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 F_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta F_\theta\right) + \frac{1}{r\sin\theta}\frac{\partial F_\phi}{\partial \phi}$$

$$\vec{\boldsymbol{\nabla}}\times\vec{F} = \frac{1}{r\sin\theta}\left(\frac{\partial}{\partial \theta}\left(\sin\theta F_\phi\right) - \frac{\partial F_\theta}{\partial \phi}\right)\hat{\boldsymbol{r}} + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial F_r}{\partial \phi} - \frac{\partial}{\partial r}\left(rF_\phi\right)\right)\hat{\boldsymbol{\theta}}$$
$$+ \frac{1}{r}\left(\frac{\partial}{\partial r}\left(rF_\theta\right) - \frac{\partial F_r}{\partial \theta}\right)\hat{\boldsymbol{\phi}}$$