

1. Water in a bathtub has velocity vector field near the drain given by

$$\vec{F} = -\frac{y+xz}{(z^2+1)^2}\hat{i} - \frac{yz-x}{(z^2+1)^2}\hat{j} - \frac{1}{z^2+1}\hat{k}$$

with  $x, y, z$  in cm, and  $\vec{F}$  in  $\frac{\text{cm}}{\text{sec}}$ .

- (a) Rewriting  $\vec{F}$  as follows, describe in words how the water is moving:

$$\vec{F} = \frac{-y\hat{i} + x\hat{j}}{(z^2+1)^2} + \frac{-z(x\hat{i} + y\hat{j})}{(z^2+1)^2} - \frac{1}{z^2+1}\hat{k}$$

- (b) The drain in the bathtub is a disk in the  $xy$ -plane with center at the origin and radius 1 cm. Find the rate at which the water is leaving the bathtub. (That is, find the rate at which water is flowing through the disk.) Give units for your answer.
- (c) Find the divergence of  $\vec{F}$ .
- (d) Find the flux of the water through the hemisphere of radius 1, centered at the origin, lying below the  $xy$ -plane and oriented downward.
- (e) Find  $\int_C \vec{G} \cdot d\vec{r}$  where  $C$  is the edge of the drain, orientated clockwise when viewed from above, and where

$$\vec{G} = \frac{1}{2} \left( \frac{y\hat{i} - x\hat{j}}{z^2+1} - \frac{x^2+y^2}{(z^2+1)^2}\hat{k} \right)$$

- (f) Calculate the curl of  $\vec{G}$ .
- (g) Explain why your answer to parts (d) and (e) are equal.