MTH 255

HW #8

1. Water in a bathtub has velocity vector field near the drain given by

$$ec{m{F}} = -rac{y+xz}{(z^2+1)^2}\, \hat{m{\imath}} - rac{yz-x}{(z^2+1)^2}\, \hat{m{\jmath}} - rac{1}{z^2+1}\, \hat{m{k}}$$

with x, y, z in cm, and \vec{F} in $\frac{\text{cm}}{\text{sec}}$.

(a) Rewriting \vec{F} as follows, describe in words how the water is moving:

$$\vec{F} = \frac{-y\,\hat{\imath} + x\,\hat{\jmath}}{(z^2+1)^2} + \frac{-z(x\,\hat{\imath} + y\,\hat{\jmath})}{(z^2+1)^2} - \frac{1}{z^2+1}\,\hat{k}$$

- (b) The drain in the bathtub is a disk in the *xy*-plane with center at the origin and radius 1 cm. Find the rate at which the water is leaving the bathtub. (That is, find the rate at which water is flowing through the disk.) Give units for your answer.
- (c) Find the divergence of \vec{F} .
- (d) Find the flux of the water through the hemisphere of radius 1, centered at the origin, lying below the *xy*-plane and oriented downward.
- (e) Find $\int_C \vec{G} \cdot d\vec{r}$ where C is the edge of the drain, orientated clockwise when viewed from above, and where

$$\vec{G} = \frac{1}{2} \left(\frac{y \,\hat{\imath} - x \,\hat{\jmath}}{z^2 + 1} - \frac{x^2 + y^2}{(z^2 + 1)^2} \,\hat{k} \right)$$

- (f) Calculate the curl of \vec{G} .
- (g) Explain why your answer to parts (d) and (e) are equal.