VECTOR DERIVATIVES

Rectangular coordinates

$$\boxed{\begin{aligned} d\vec{\boldsymbol{r}} &= dx\,\hat{\boldsymbol{\imath}} + dy\,\hat{\boldsymbol{\jmath}} + dz\,\hat{\boldsymbol{k}} \\ \vec{\boldsymbol{F}} &= F_x\,\hat{\boldsymbol{\imath}} + F_y\,\hat{\boldsymbol{\jmath}} + F_z\,\hat{\boldsymbol{k}} \end{aligned}}$$

$$\begin{aligned} \vec{\boldsymbol{\nabla}} f &= \frac{\partial f}{\partial x}\,\hat{\boldsymbol{\imath}} + \frac{\partial f}{\partial y}\,\hat{\boldsymbol{\jmath}} + \frac{\partial f}{\partial z}\,\hat{\boldsymbol{k}} \\ \vec{\boldsymbol{\nabla}} \cdot \vec{\boldsymbol{F}} &= \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} \\ \vec{\boldsymbol{\nabla}} \times \vec{\boldsymbol{F}} &= \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\hat{\boldsymbol{\imath}} + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right)\hat{\boldsymbol{\jmath}} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\hat{\boldsymbol{k}} \end{aligned}$$

Cylindrical coordinates

$$\boxed{\begin{aligned} d\vec{\boldsymbol{r}} &= dr\,\hat{\boldsymbol{r}} + r\,d\phi\,\hat{\boldsymbol{\phi}} + dz\,\hat{\boldsymbol{z}} \\ \vec{\boldsymbol{F}} &= F_r\,\hat{\boldsymbol{r}} + F_\phi\,\hat{\boldsymbol{\phi}} + F_z\,\hat{\boldsymbol{z}} \end{aligned}}$$

$$\begin{aligned} \vec{\boldsymbol{\nabla}} f &= \frac{\partial f}{\partial r}\,\hat{\boldsymbol{r}} + \frac{1}{r}\frac{\partial f}{\partial \phi}\,\hat{\boldsymbol{\phi}} + \frac{\partial f}{\partial z}\,\hat{\boldsymbol{z}} \\ \vec{\boldsymbol{\nabla}} \cdot \vec{\boldsymbol{F}} &= \frac{1}{r}\frac{\partial}{\partial r}(rF_r) + \frac{1}{r}\frac{\partial F_\phi}{\partial \phi} + \frac{\partial F_z}{\partial z} \\ \vec{\boldsymbol{\nabla}} \times \vec{\boldsymbol{F}} &= \left(\frac{1}{r}\frac{\partial F_z}{\partial \phi} - \frac{\partial F_\phi}{\partial z}\right)\hat{\boldsymbol{r}} + \left(\frac{\partial F_r}{\partial z} - \frac{\partial F_z}{\partial r}\right)\hat{\boldsymbol{\phi}} + \frac{1}{r}\left(\frac{\partial}{\partial r}(rF_\phi) - \frac{\partial F_r}{\partial \phi}\right)\hat{\boldsymbol{z}} \end{aligned}$$

Spherical coordinates

$$\boxed{\begin{aligned} d\vec{\boldsymbol{r}} &= dr\,\hat{\boldsymbol{r}} + r\,d\theta\,\hat{\boldsymbol{\theta}} + r\,\sin\theta\,d\phi\,\hat{\boldsymbol{\phi}} \\ \vec{\boldsymbol{F}} &= F_r\,\hat{\boldsymbol{r}} + F_\theta\,\hat{\boldsymbol{\theta}} + F_\phi\,\hat{\boldsymbol{\phi}} \end{aligned}}$$

$$\begin{aligned} \vec{\boldsymbol{\nabla}} f &= \frac{\partial f}{\partial r}\,\hat{\boldsymbol{r}} + \frac{1}{r}\frac{\partial f}{\partial \theta}\,\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\,\hat{\boldsymbol{\phi}} \\ \vec{\boldsymbol{\nabla}} \cdot \vec{\boldsymbol{F}} &= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 F_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta F_\theta) + \frac{1}{r\sin\theta}\frac{\partial F_\phi}{\partial \phi} \\ \vec{\boldsymbol{\nabla}} \times \vec{\boldsymbol{F}} &= \frac{1}{r\sin\theta}\left(\frac{\partial}{\partial \theta}(\sin\theta F_\phi) - \frac{\partial F_\theta}{\partial \phi}\right)\hat{\boldsymbol{r}} + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial F_r}{\partial \phi} - \frac{\partial}{\partial r}(rF_\phi)\right)\hat{\boldsymbol{\theta}} \\ &\quad + \frac{1}{r}\left(\frac{\partial}{\partial r}(rF_\theta) - \frac{\partial F_r}{\partial \theta}\right)\hat{\boldsymbol{\phi}} \end{aligned}$$
