

Subtract!

• sphere

$$\begin{aligned} & \int_0^{2\pi} \int_0^{\pi} \int_0^{\sqrt{2}} r^2 \sin^2 \theta \cdot r^2 \sin \theta \, dr \, d\theta \, d\phi \\ &= 2\pi \frac{r^5}{5} \Big|_0^{\sqrt{2}} \int_0^{\pi} (1 - \cos^2 \theta) \sin \theta \, d\theta \\ &= \frac{2\pi \sqrt{32}}{5} \left(-\cos \theta + \frac{\cos^3 \theta}{3} \right) \Big|_0^{\pi} = \frac{8\pi \sqrt{32}}{15} = \underline{\underline{\frac{32\pi \sqrt{2}}{15}}} \end{aligned}$$

• cylinder

$$\int_{-1}^1 \int_0^{2\pi} \int_0^1 r^2 r \, dr \, d\phi \, dz = 2\pi \cdot \frac{r^4}{4} \Big|_0^1 \cdot z \Big|_{-1}^1 = \underline{\underline{\pi}}$$

• 2 caps

$$\begin{aligned} & 2 \int_1^{\sqrt{2}} \int_0^{2\pi} \int_0^{\sqrt{2-z^2}} r^2 r \, dr \, d\phi \, dz \\ &= 4\pi \int_1^{\sqrt{2}} \frac{r^4}{4} \Big|_0^{\sqrt{2-z^2}} dz \\ &= \pi \int_1^{\sqrt{2}} (2-z^2)^2 dz \\ &= \pi \left(4z - \frac{4z^3}{3} + \frac{z^5}{5} \right) \Big|_1^{\sqrt{2}} = \underline{\underline{\pi \left(\frac{32\sqrt{2}}{15} - \frac{43}{15} \right)}} \end{aligned}$$

$$\therefore \frac{32\pi}{15} \sqrt{2} - \pi - \frac{32\pi \sqrt{2}}{15} + \frac{43\pi}{15} = \boxed{\frac{28\pi}{15}}$$